Design

Memo 22a

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Reinforcement design for BSF 150/20

PART 1 – BASIC ASSUMPTIONS

1.1 GENERAL

In these calculations certain assumptions has been made about dimensions and qualities in the precast concrete elements that may not always be the case. Therefore the following calculations of anchorage of the units and the resulting reinforcement must be considered as an example to illustrate the calculation model.

In beams it must always be checked that the forces from the anchorage reinforcement can be transferred to the beam's main reinforcement. The recommended shear reinforcement (stirrups) includes all necessary stirrups in the beam end; i.e. the normal shear reinforcement in beam ends and an addition due to the cantilever action of the beam unit from the beam.

The information found here and in the memos assumes that the design of the elements and the use of the units in structural elements are carried out under the supervision of a structural engineer with knowledge about the behaviour of concrete structures.

1.2 STANDARDS

The calculations are carried out according to:

Eurocode 2: Design of concrete structures. Part 1-1. General rules and rules for buildings.

Eurocode 3: Design of steel structures. Part 1-1: General rules and rules for buildings.

Eurocode 3: Design of steel structures. Part 1-8: Design of joints.

EN 10080: Steel for the reinforcement of concrete. Weldable reinforcing steel. General.

For all NPDs in the Eurocodes the recommended values are used.

These NDPs are: In EC 2: γ_c; γ_s; α_{cc}; α_{ct}; k_1; k_2 and $\varnothing_{m,min}$. In EC 3: γ_{M0}; γ_{M1} and γ_{M2}.

1.3 LOADS

Vertical ultimate limit state load = F_V = 200 kN.

Horizontal ultimate limit state load = F_H = 0,3×F_V = 60 kN.

The design is carried out with a horizontal force of 30 % of the vertical force. Recommended capacity for transfer of active forces (as wind loads) may be assumed to be 20 % of smallest vertical load present at the same time.

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1.4 QUALITIES

Concrete grade C45/55:

$$f_{cd} = \alpha_{cc} \cdot f_{ck}/\gamma_c = 1{,}0 \cdot 45/1{,}5 = 30{,}0 \text{ MPa}$$

$$f_{ctd} = \alpha_{ct} \cdot f_{ctk,0,05}/\gamma_c = 1{,}0 \cdot 2{,}70/1{,}5 = 1{,}80 \text{ MPa}$$

$$f_{bd} = 2{,}25 \cdot \eta_1 \cdot \eta_2 \cdot f_{ctd} = 2{,}25 \cdot 1{,}0 \cdot 1{,}0 \cdot 1{,}80 = 4{,}05 \text{ MPa}$$

Reinforcement B500C:

$$f_{yd} = f_{yk}/\gamma_s = 500/1{,}15 = 435 \text{ MPa}$$

Structural steel S355:

$$f_{yd} = f_y/\gamma_{M0} = 355/1{,}00 = 355 \text{ MPa} < f_u/\gamma_{M2} = 510/1{,}25 = 408 \text{ MPa}$$

Shear: $\quad f_{sd} = f_y/(\gamma_{M0} \cdot \sqrt{3}) = 355/(1{,}00 \cdot \sqrt{3}) = 205 \text{ MPa}$

Welds: $\quad f_{vw.d} = \dfrac{f_u}{\gamma_{M2}\sqrt{3}} \times \dfrac{1}{\beta_w} = \dfrac{510}{1{,}25 \times \sqrt{3}} \times \dfrac{1}{0{,}9} = 262 \text{ MPa}$

Half round steel S275 N/NL:

$$f_{yd} = f_y/\gamma_{M0} = 275/1{,}00 = 275 \text{ MPa} < f_u/\gamma_{M2} = 430/1{,}25 = 344 \text{ MPa}$$

Welds: $\quad f_{w,d} = \dfrac{f_u}{\gamma_{M2}\sqrt{3}} \times \dfrac{1}{\beta_w} = \dfrac{430}{1{,}25 \times \sqrt{3}} \times \dfrac{1}{0{,}85} = 233 \text{ MPa}$

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PART 2 – ANCHORAGE OF THE UNITS

2.1 BEAM UNIT – EQUILIBRIUM

Figure 1. Designations and assumed dimensions.

Neglecting the moment created by the small vertical shift in the transfer of F_H.

$R_{CU} = F_V \times d_1/d_2$

$R_{CO} = F_V + R_{CU}$

$l_{CU} = R_{CU} /(f_{cd} \times b)$ (b is the width of the beam unit = 36 mm.)

$l_{CO}/2$ is decided by the location of the front reinforcement. Assume 2-Ø16 welded to the front plate plus 2-Ø10 bent over the half round steel:

$$l_{CO}/2 = \frac{201 \times 22 + 78,5 \times (52 + 82)}{201 + 2 \times 78,5} = 42 \text{ mm}$$

$d_1 = a_1 - a_2 + l_{CO}/2$

$d_2 = a_2 - l_{CO}/2 - l_{CU}/2$

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Use a spread sheet. Assume d_1/d_2, calculate resulting d_1/d_2. Change assumed value until they are equal. (Spread sheet "equlibrim-BSF150-20-beamunit.exc".)

Equlibrium of BSF 150/20 beam unit			
		Concrete grade = C	45 /55
		Material coefficient for concrete =	1,5
Geometry:	a_1 = 465 mm	f_{cd} =	30,0 MPa
	a_2 = 400 mm		
Width of beam unit =	100 mm	F_V =	200 kN
$l_{CO}/2$ =	42 mm		
		Assumed d_1 / d_2 =	0,3076982
R_{CU} = 62 kN		d_1 =	107 mm
R_{CO} = 262 kN		d_2 =	348 mm
l_{CU} = 21 mm			
		Calculated d_1 /d_2 =	0,3076982

2.2 BEAM UNIT – ANCHORAGE IN FRONT

2.2.1 Check of capacity

$A_{s,req'd} = R_{CO}/f_{sd,reinf} = 262/0,435 = 602$ mm^2

Assumed 2-Ø16 + 2-Ø10 = 2×201+2×2×78,5 = 716 mm^2 – ok

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2.2.2   Anchorage

As illustrated in figure 2 the reinforcement will be very congested when the beam has minimum dimensions. This must be taken into consideration when calculating the anchorage length (the horizontal part of the front reinforcement). Since the reinforcing bars are so close together this may necessitate considering them as bundled bars. (The main reinforcement in the beam is not shown on the side view.)

According to EC2 clauses 8.4.3 and 8.4.4:

$l_{bd} = \alpha_1 \cdot \alpha_2 \cdot \alpha_3 \cdot \alpha_4 \cdot \alpha_5 \cdot l_{b,reqd} \geq l_{b,min}$

$l_{b,reqd} = \dfrac{\varnothing}{4} \cdot \dfrac{\sigma_{sd}}{f_{bd}} = \dfrac{16}{4} \cdot \dfrac{262/0{,}716}{4{,}05} = 361$ mm

$l_{b,min} = \max(0{,}3 \cdot l_{b,reqd};\ 10 \cdot \varnothing;\ 100$ mm$)$

   $= 160$ mm

Straight bar:

   $\alpha_1 = 1{,}0$

Concrete cover:

   $c_d = \min(a/2;\ c_1;\ c)$

   $= \min(20/2;\ 50;\ 40) = 10$ mm

   $\alpha_2 = 1-0{,}15 \cdot (c_d-3 \cdot \varnothing)/\varnothing$

   $= 1-0{,}15 \cdot (10-3 \cdot 16)/16 > 1{,}0$

   $\alpha_2 = 1{,}0$

2-Ø10 stirrups bent over half round steel

2-Ø16 welded to front plate

200 (minimum)

400 (minimum)
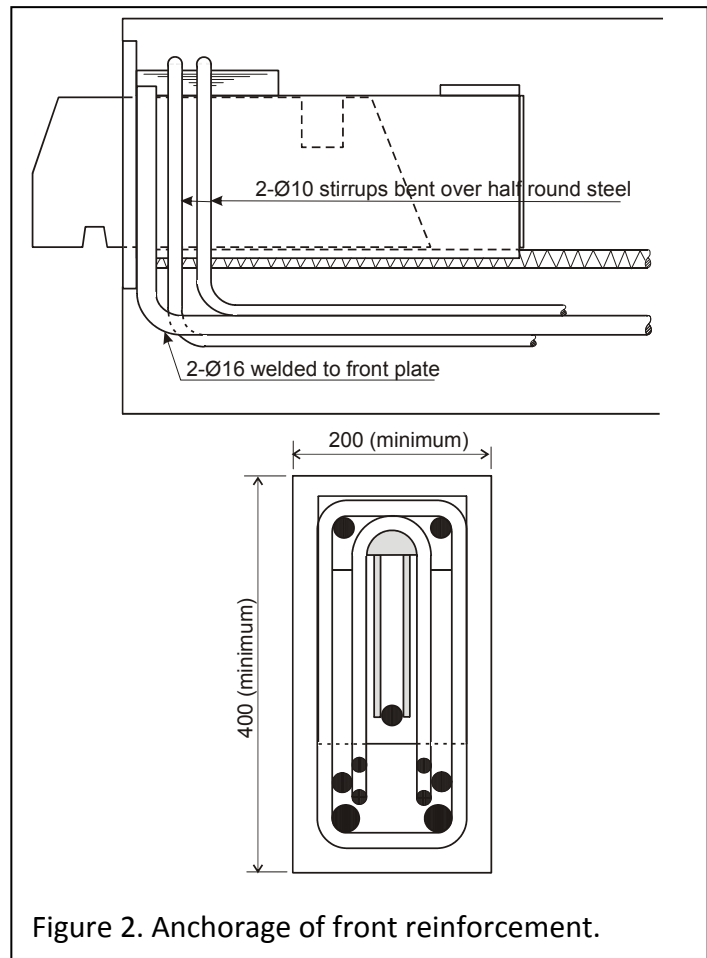
Figure 2. Anchorage of front reinforcement.

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Confinement by reinforcement: Assume transverse reinforcement Ø12-c/c150 mm:

$$\alpha_3 = 1 - K \cdot \lambda = 1 - 0,05 \cdot \frac{113 \cdot \dfrac{400}{150} - 0,25 \cdot 201}{201} = 0,94$$

Confinement by welded transverse reinforcement:

$\alpha_4 = 1,0$

Confinement by transverse pressure:

$\alpha_5 = 1,0$

$l_{bd} = 1,0 \cdot 1,0 \cdot 0,94 \cdot 1,0 \cdot 1,0 \cdot 361 = 339$ mm

Required welding of reinforcing bars is shown in clause 2.5.

It must always be checked that the beam's main reinforcement has sufficient anchorage at the end of the horizontal part of the front anchorage. This will probably lead to greater lengths for the horizontal part of the front anchorage than calculated here.

The force is approximately $[F_V \times \dfrac{a}{z} + \dfrac{F_V}{2} + F_H \dfrac{x_1}{z}]$. Additional force in the beam's top reinforcement is about $[F_H \cdot \dfrac{x_2}{z}]$, but this force leads to a reduction in the compressive stresses and may be neglected. If there is a large distance between the horizontal part of the front anchorage and the beam's main reinforcement, the length of the horizontal part of the front anchorage must be increased with this distance. (This distance is very small in the figure.)
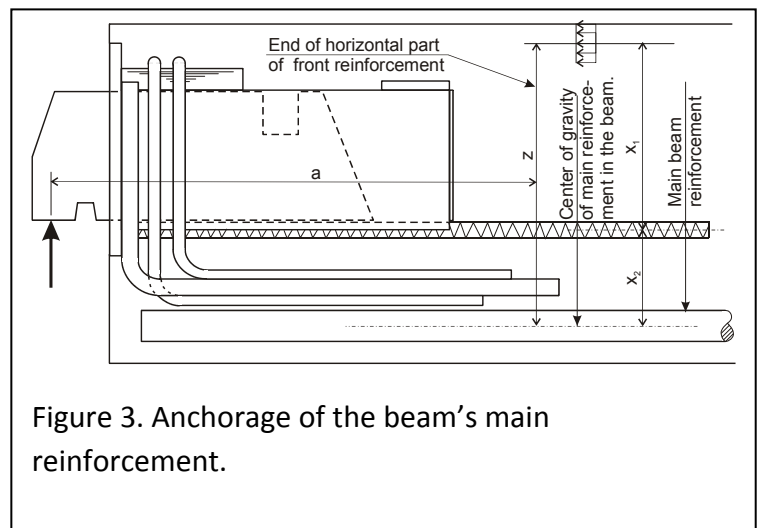

Figure 3. Anchorage of the beam's main reinforcement.

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2.2.3 Bending of front reinforcement and requirements for transverse reinforcement

See figure 4. Stress in the anchorage reinforcement is 262/0,716 = 366 MPa

According to EN 1992-1-1, clause 8.3:

$Ø_{m,min}$ is generally $7 \cdot 10 = 70$ mm and $7 \cdot 16 = 112$ mm to avoid damage to the reinforcing bars.

To avoid cross bars:

$$Ø_{m,min} = \frac{F_{bt} \cdot (\tfrac{1}{a_b} + \tfrac{1}{2} \cdot Ø)}{f_{cd}}$$

Ø10 bars:

$$Ø_{m,min} = \frac{262 \cdot 10^3}{30} \cdot \left(\frac{1}{32} + \frac{1}{2 \cdot 10} \right) = 710 \text{mm}$$

Ø16 bars:

$$Ø_{m,min} = \frac{262 \cdot 10^3}{30} \cdot \left(\frac{1}{67} + \frac{1}{2 \cdot 16} \right) = 403 \text{mm}$$

Chosen reference depth

Beam depth

Height of beam unit

Ø$_m$/2   Minimum 5 mm

Ø$_m$/2   Minimum distance = required cover plus stirrup diameter

Diameter of reinforcement

Figure 4. Bending of front reinforcement.

If cross bars are required the total area of this reinforcement shall be at least 40% of the area of the bent bars. (This last requirement is from the Norwegian concrete design code, as EC2 does not contain any particular requirements for such reinforcement.) It is the opinion of this structural engineer that it may be reduced according to the stress in the anchorage reinforcement. The cross bars shall consist of at least two bars placed inside the bend. The cross bars can be anchored with 90º hook at the ends or by being shaped like a U-bar.
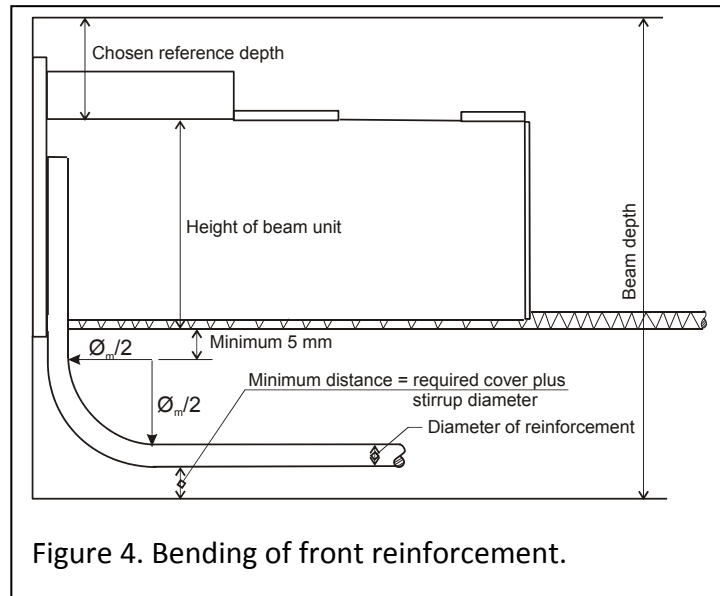
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Bar diameter	Mandrel diameter	Transverse reinforcement
10	≥ 710 mm	Not required
	< 710 mm; ≥ 70 mm	Required
16	≥ 403 mm	Not required
	< 403 mm; ≥ 112 mm	Required

Assumed concrete grade is C45/55. For other concrete grades and bar diameters the mandrel diameters will be different.

2.2.4 Minimum beam depth

See figure 4 and Memo 10.

The recommended reference depth is 80 mm and the height of the beam unit is 170 mm. Assume that Ø12 stirrups will require 15 mm space, and assume 25 mm concrete cover. Also assume that a 16 mm bar requires 20 mm space.

To avoid cross bars the minimum beam depth will be 80+170+5+(710/2)+20+15+25 = 670 mm ≈ 700 mm.

Accepting cross bars the minimum beam depth will be 80+170+5+(112/2)+20+15+25 = 371 mm ≈ 400 mm.

If cross bars are required: $0{,}4 \cdot 716 \cdot 366/435 = 242$ mm^2

If U-shaped bars are used they will be properly anchored in one end only. Hence the total reinforcement area must be twice of the calculated area. Each bar has two legs:

Select two Ø10 U-shaped stirrups inserted from each side:

$2 \cdot 2 \cdot 78{,}5 \cdot 2 = 628$ mm^2 > $242 \cdot 2 = 484$ mm^2.

2.3 BEAM UNIT – ANCHORAGE OF HORIZONTAL FORCES

2.3.1 Check of capacity

$F_H = 60$ kN (see clause 1.3)

$A_{s,nødv} = 60/0{,}435 = 138$ mm^2

1-Ø16 is assumed (201 mm^2) - ok

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2.3.2 Anchorage length

According to EC2 clause 8.4.4:

$l_{bd} = \alpha_1 \cdot \alpha_2 \cdot \alpha_3 \cdot \alpha_4 \cdot \alpha_5 \cdot l_{b,reqd} \geq l_{b,min}$

$$l_{b,reqd} = \frac{\varnothing}{4} \cdot \frac{\sigma_{sd}}{f_{bd}} = \frac{16}{4} \cdot \frac{60/0,201}{4,05} = 295 \text{ mm}$$

$l_{b,min} = \max(0,3 \cdot l_{b.reqd}; \ 10 \cdot \varnothing; \ 100 \text{ mm}) = 160 \text{ mm}$

Straight bar:

$\alpha_1 = 1,0$

Concrete cover:

$c_d = \min(a/2; \ c_1; \ c) = \min(42/2; \ 50; \ 40) = 21 \text{ mm}$

$\alpha_2 = 1 - 0,15 \cdot (c_d - 3 \cdot \varnothing)/\varnothing = 1 - 0,15 \cdot (21 - 3 \cdot 16)/16 > 1,0 \ ; \text{ i.e. } \alpha_2 = 1,0$

Confinement by reinforcement: Assume transverse reinforcement $\varnothing$12-c/c150 mm:

$$\alpha_3 = 1 - K \cdot \lambda = 1 - 0,05 \cdot \frac{113 \cdot \frac{400}{150} - 0,25 \cdot 201}{201} = 0,94$$

Confinement by welded transverse reinforcement:

$\alpha_4 = 1,0$

Confinement by transverse pressure:

$\alpha_5 = 1,0$

$l_{bd} = 1,0 \cdot 1,0 \cdot 0,94 \cdot 1,0 \cdot 1,0 \cdot 295 = 277 \text{ mm}$

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2.4 COLUMN UNIT – ANCHORAGE IN THE COLUMN

2.4.1 Check of capacity

$M = F_H \times (10 + 15/2) + F_V \times \{[(75+12)/2] - 42\} = 200 \times (0,3 \times 17,5 + 1,5) = 1350$ kNmm

$S = 1350/204 = 6,62$ kN

$A_s = 6,62/0,435 = 15$ mm^2

1-Ø8 at the top is sufficient.

$F_H - S = 0,3 \times 200 - 7 = 53$ kN

$A_s = 53/0,435 = 122$ mm^2

2-Ø10 at the bottom is sufficient.

Figure 5. Assumed dimensions. Equilibrium of the column unit.

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2.4.2 Anchorage of welded reinforcement

Normally there are similar units on opposite sides of the column. The easiest anchorage is then to connect the units as shown in Memo 43.

If anchorage bars bent up and down are used, the amount of reinforcement must be as calculated in clause 2.4.1.

If U-shaped bars are used the area of the reinforcing bar may be half of that calculated in clause 2.4.1, because two sections are providing the anchorage. It will improve the anchorage considerably if a bar is placed in the bend of the U-shaped bars.

Figure 6. Methods for anchorage of the column unit.

Necessary welding of reinforcing bars is shown in clause 2.5.

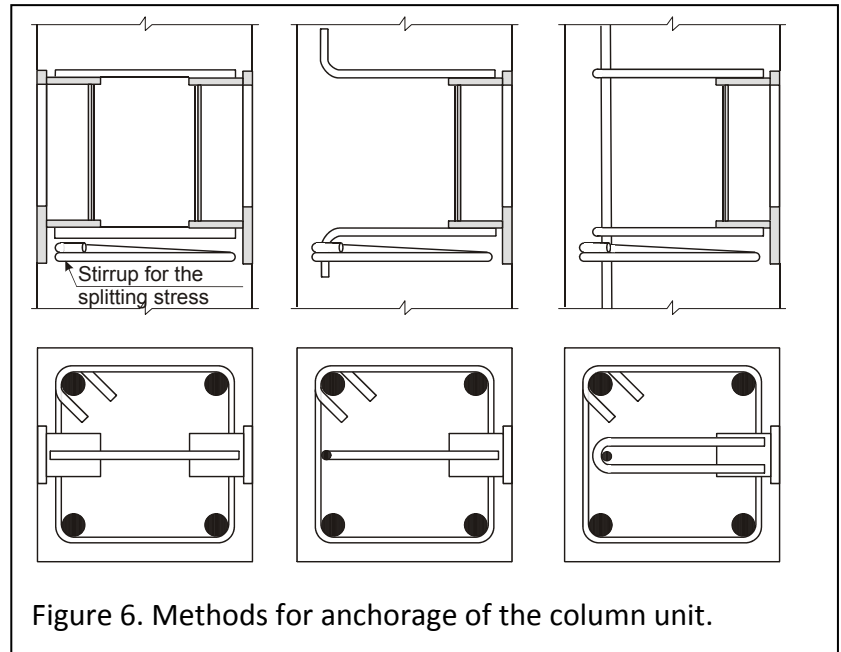The stirrup for the splitting stress is calculated in clause 3.4.

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2.5   WELDING OF REINFORCEMENT TO THE UNITS

Selecting to always weld for the capacity of the bar.

Assuming welds on both sides of the bar. Length of weld = l_w.

For reinforcing bar Ø16:

$l_w \cdot a = (201 \cdot 0{,}435)/(0{,}262 \cdot 2) = 167$ mm^2

Select a = 4 mm;   i.e. $l_{w,min} = 42$ mm

For reinforcing bar Ø20

$l_w \cdot a = (314 \cdot 0{,}435)/(0{,}262 \cdot 2) = 261$ mm^2

Select a = 5 mm;   i.e. $l_{w,min} = 52$ mm

For reinforcing bar Ø25:

$l_w \cdot a = (491 \cdot 0{,}435)/(0{,}262 \cdot 2) = 408$ mm^2

Select a = 5 mm;   i.e. $l_{w,min} = 82$ mm

For reinforcing bar Ø32:

$l_w \cdot a = (804 \cdot 0{,}435)/(0{,}262 \cdot 2) = 667$ mm^2

Select a = 6 mm;   i.e. $l_{w,min} = 111$ mm

Figure 7. Welding of reinforcing bars.
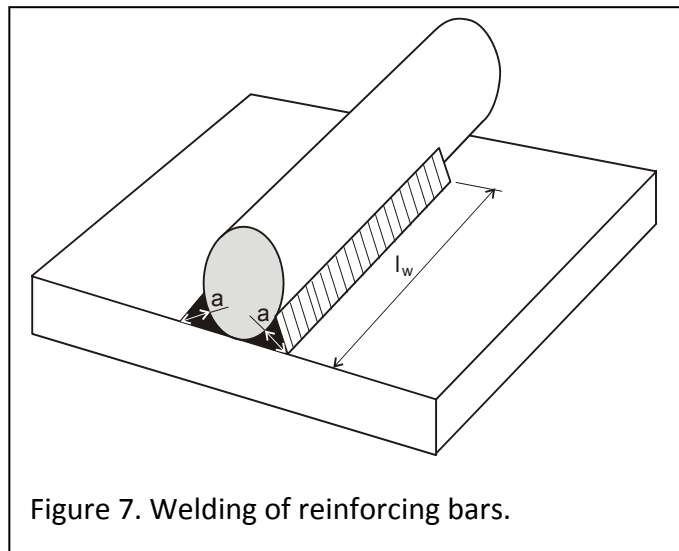
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PART 3 – REINFORCEMENT

3.1 BEAM – REINFORCEMENT IN THE REAR

$A_{s,reqd} = R_{CU}/f_{sd} = 62/0{,}435 = 143$ mm^2,

corresponding to $143/(2{\times}78{,}5) = 0{,}91$; i.e. $1 - \varnothing 10$ stirrup.

3.2 BEAM – SHEAR STIRRUPS

Use a strut-and-tie model with compressive diagonals at 45º.

The shear force within the central part of the beam unit is assumed to be $R_{CO} = 262$ kN.

Assume $z = 300$ mm and stirrup diameter $\varnothing 10$.

$s = A_{sw}{\cdot}z{\cdot}f_{yd}{\cdot}\cot \theta/V$

$s = 2{\times}78{,}5{\times}435{\times}300{\times}\cot 45º / 262{\times}10^3 = 70$ mm

Select $\varnothing 10$ c/c 70 mm. This reinforcement should be brought approximately 200 mm past the end of the beam unit in order to absorb any splitting effects from the anchorage of reinforcement in the rear.

If $\varnothing 12$ stirrups are selected the center distance will be 110 mm

3.3 BEAM – CHECK OF SHEAR COMPRESSION

Assume beam width of 300 mm and beam depth of 450 mm. (See also clause 2.2.) This means that $z_{min} \approx$ 450−50−100 = 300 mm with one layer of reinforcement in the top of the beam and three layers in the bottom.

The shear reinforcement is calculated based on 45º compressive field. The check of shear compression will be carried out under the same assumption:

$V_{Rd,max} = \alpha_{cw}{\cdot}b_w{\cdot}z{\cdot}\upsilon_1{\cdot}f_{cd}/(\cot \theta + \tan \theta)$

$V_{Rd,max} = \{1{,}0{\cdot}(300{-}100){\cdot}300{\cdot}0{,}6{\cdot}[1{-}(45/250)]{\cdot}30{,}0/(\cot 45º{+}\tan 45º)\}{\cdot}10^{-3}$

$V_{ccd} = 442$ kN > 262 kN - OK

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3.4   COLUMN – SPLITTING STRESS

Splitting stress under the column unit according to Leonhardt $\approx 0{,}2 \times F_V = 0{,}2 \times 200 = 40$ kN

$A_S = 40/0{,}435 = 92$ mm^2

1-Ø8 stirrup ($2 \times 50{,}3$ mm^2) is sufficient.

3.5   CONCLUSION – REINFORCEMENT IN THE BEAM

For clarity the beam's main reinforcement is not shown in the side view. Between the shown stirrups in each end of the beam a normal calculation of the shear reinforcement must be carried out. The main reinforcement in the beam must of course also be calculated.

Figure 8. Reinforcement in the beam end.
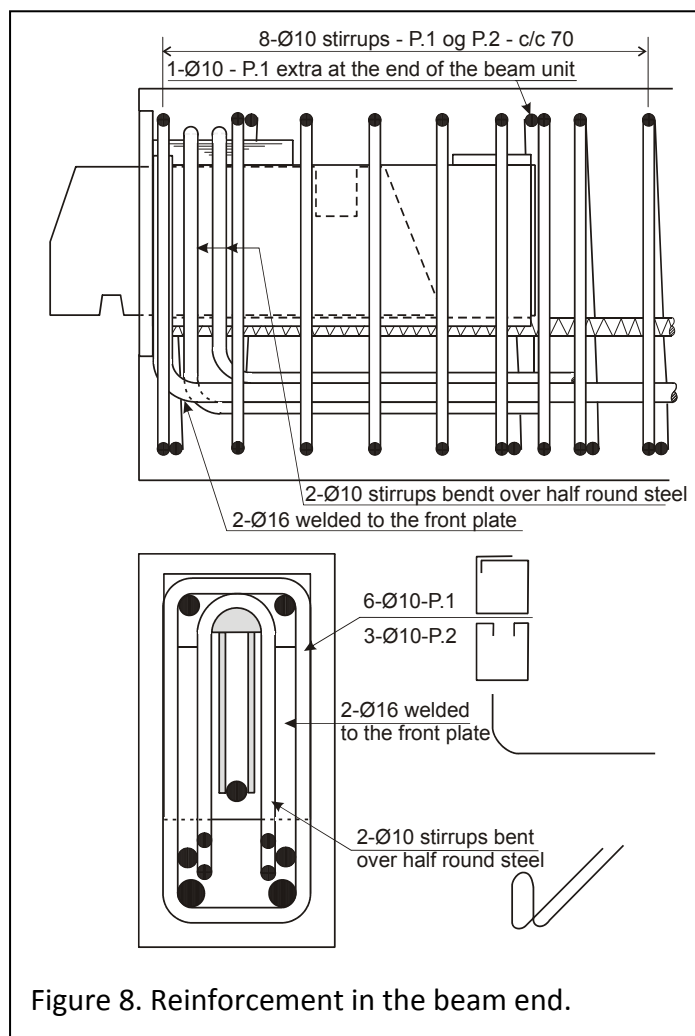
At the end of the front anchorage it must be checked that the beam's main reinforcement has sufficient anchorage. See clause 2.2.4.