

Date :	18.10.05	Sign: tb
Last rev:	26.09.11	Sign: tb
Doc. No:	K4-10/39bE	Sign: tb
Page 1 of 12		

Reinforcement design for BCC 450

PART 1 – BASIC ASSUMPTIONS

1.1 GENERAL

In these calculations certain assumptions has been made about dimensions and qualities in the precast concrete elements that may not always be the case. Therefore the following calculations of anchorage of the units and the resulting reinforcement must be considered as an example to illustrate the calculation model.

In beams it must always be checked that the forces from the anchorage reinforcement can be transferred to the beam's main reinforcement. The recommended shear reinforcement (stirrups) includes all necessary stirrups in the beam end; i.e. the normal shear reinforcement in beam ends and an addition due to the cantilever action of the beam unit from the beam.

The information found here and in the memos assumes that the design of the elements and the use of the units in structural elements are carried out under the supervision of a structural engineer with profound knowledge of the behaviour of concrete structures.

1.2 STANDARDS

The calculations are carried out according to:

Eurocode 2: Design of concrete structures. Part 1-1. General rules and rules for buildings.

Eurocode 3: Design of steel structures. Part 1-1: General rules and rules for buildings.

Eurocode 3: Design of steel structures. Part 1-8: Design of joints.

EN 10080: Steel for the reinforcement of concrete. Weldable reinforcing steel. General.

For all NPDs in the Eurocodes the recommended values are used.

These NDPs are: In EC 2: γ_c ; γ_s ; α_{cc} ; α_{ct} ; k_1 ; k_2 and $\phi_{m,min}$. In EC 3: γ_{M0} ; γ_{M1} and γ_{M2} .

1.3 LOADS

Vertical ultimate limit state load = $F_V = 450$ kN

Horizontal ultimate limit state load = $F_H = 0,3 \times F_V = 135$ kN

The design is carried out with a horizontal force of 30 % of the vertical force. Recommended capacity for transfer of active forces (as wind loads) may be assumed to be 20 % of smallest vertical load present at the same time.

Date :	18.10.05	Sign: tb
Last rev:	26.09.11	Sign: tb
Doc. No:	K4-10/39bE	Sign: tb
Page 2 of 12		

Reinforcement design for BCC 450

1.4 QUALITIES

Concrete grade C45/55: $f_{cd} = \alpha_{cc} \cdot f_{ck} / \gamma_c = 1,0 \cdot 45 / 1,5 = 30,0 \text{ MPa}$

$$f_{ctd} = \alpha_{ct} \cdot f_{ctk,0,05} / \gamma_c = 1,0 \cdot 2,70 / 1,5 = 1,80 \text{ MPa}$$

$$f_{bd} = 2,25 \cdot \eta_1 \cdot \eta_2 \cdot f_{ctd} = 2,25 \cdot 1,0 \cdot 1,0 \cdot 1,80 = 4,05 \text{ MPa}$$

Reinforcement B500C: $f_{yd} = f_{yk} / \gamma_s = 500 / 1,15 = 435 \text{ MPa}$

Tension in threaded bars: 8.8 quality steel: $f_{yd} = f_y / \gamma_{M2} = 640 / 1,25 = 512 \text{ MPa}$

Nominal diameter (mm)	M10	M12	M16	M20	M24	M30	M33	M36
Equivalent diameter (mm)	8,6	10,4	14,1	17,7	21,2	26,7	29,7	32,6
Stress area (mm ²)	58	84	157	245	353	561	694	835

Structural steel S355:

Tension and compression:

$$f_{yd} = f_y / \gamma_{M0} = 355 / 1,00 = 355 \text{ MPa} < f_u / \gamma_{M2} = 510 / 1,25 = 408 \text{ MPa}$$

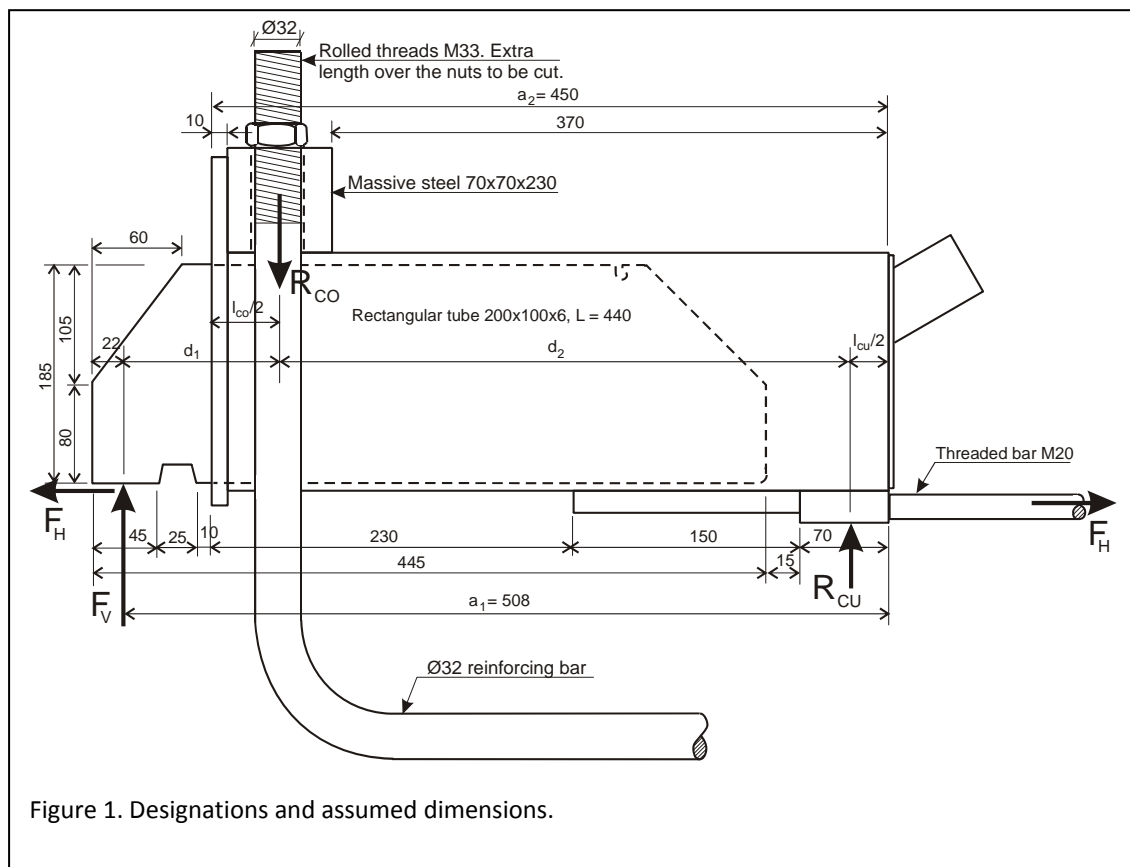
Shear: $f_{sd} = f_y / (\gamma_{M0} \cdot \sqrt{3}) = 355 / (1,00 \cdot \sqrt{3}) = 205 \text{ MPa}$

$$\text{Welds: } f_{vw,d} = \frac{f_u}{\gamma_{M2} \sqrt{3}} \times \frac{1}{\beta_w} = \frac{510}{1,25 \times \sqrt{3}} \times \frac{1}{0,9} = 262 \text{ MPa}$$

Reinforcement design for BCC 450

PART 2 – ANCHORAGE OF THE UNITS

2.1 BEAM UNIT - EQUILIBRIUM



Neglecting the moment created by the small vertical shift in the transfer of F_H .

$$R_{CU} = F_V \times d_1 / d_2$$

$$R_{CO} = F_V + R_{CU}$$

$$l_{CU} = R_{CU} / (f_{cd} \times b) \text{ (b is the width of the beam unit)}$$

$$l_{CO}/2 \text{ is decided geometrically: } l_{CO}/2 = 10 + 70/2 = 45 \text{ mm}$$

$$d_1 = a_1 - a_2 + l_{CO}/2$$

$$d_2 = a_2 - l_{CO}/2 - l_{CU}/2$$

Reinforcement design for BCC 450

Use a spread sheet. Assume d_1/d_2 , calculate resulting d_1/d_2 . Change assumed value until they are equal. (Spread sheet "Equilibrium-BCC450-beam-unit.exc".)

Equilibrium of BCC 450 beam unit:	
Concrete grade = C 45 /55	
Material coefficient for concrete = 1,5	
Geometry:	$a_1 = 508$ mm $f_{cd} = 30,0$ MPa
	$a_2 = 450$ mm
Width of the beam unit =	100 mm $F_V = 450$ kN
$l_{CO}/2 =$	45 mm
Assumed $d_1 / d_2 = 0,2675801$	
$R_{CU} = 120$ kN	$d_1 = 103$ mm
$R_{CO} = 570$ kN	$d_2 = 385$ mm
$l_{CU} = 40$ mm	
Calculated $d_1 / d_2 = 0,2675801$	

Date :	18.10.05	Sign: tb
Last rev:	26.09.11	Sign: tb
Doc. No:	K4-10/39bE	Sign: tb
Page 5 of 12		

Reinforcement design for BCC 450

2.2 BEAM UNIT – ANCHORAGE IN FRONT

2.2.1 Check of capacity

$$A_{s,reqd} = R_{CO}/f_{yd, reinf} = 570/0,435 = 1310 \text{ mm}^2$$

$$\text{Assumed } 2-\text{Ø}32 = 2 \times 804 = 1608 \text{ mm}^2 - \text{ok}$$

2.2.2 Bending of front reinforcement

See figure 2. Stress in the anchorage reinforcement = $570 \cdot 10^3 / 1608 = 355 \text{ MPa}$

According to EN 1991-1-1, clause 8.3:

$\text{Ø}_{m, \min}$ is generally $7 \cdot 32 = 224 \text{ mm}$ to avoid damage to the reinforcing bars.

To avoid cross bars:

$$\text{Ø}_{m, \min} = \frac{F_{bt} \cdot \left(\frac{1}{a_b} + \frac{1}{2} \cdot \phi \right)}{f_{cd}}$$

$$\text{Ø}_{m, \min} = \frac{570 \cdot \frac{804}{1608} \cdot 10^3}{30} \cdot \left(\frac{1}{140/2} + \frac{1}{2 \cdot 32} \right) = 284 \text{ mm}$$

For mandrel diameters equal to or larger than 284 mm there is no requirement to transverse reinforcement in the bend. Small distance from the side of the beam to the anchorage bars may still make cross bars necessary.

With concrete grade C45/55 transverse reinforcement in the bend will be required for

$$224 \text{ mm} \leq \text{Ø}_m \leq 284 \text{ mm}.$$

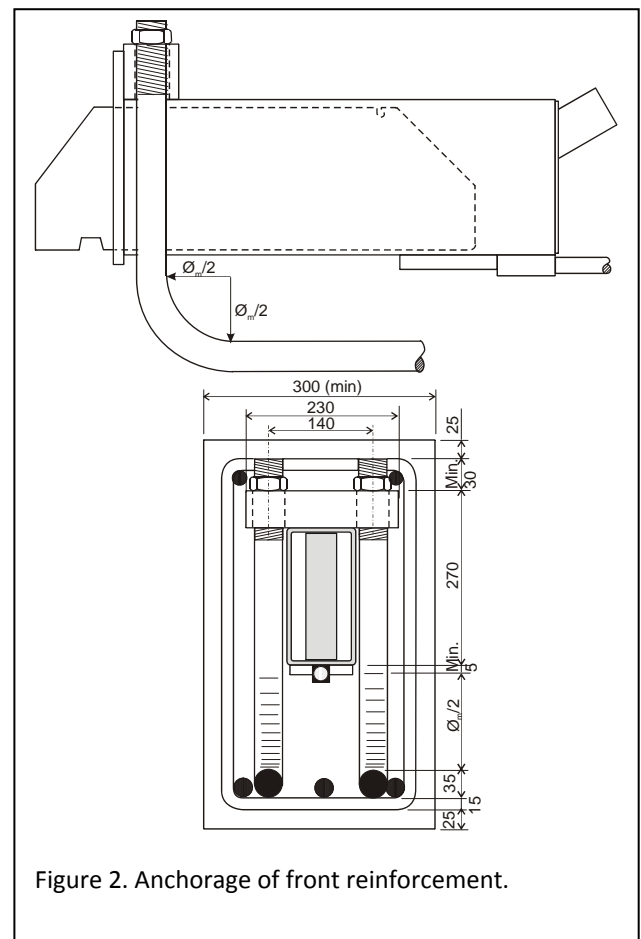


Figure 2. Anchorage of front reinforcement.

For other concrete grades and bar diameters the mandrel diameters will be different.

If cross bars are required the total area of this reinforcement shall be at least 40% of the area of the bent bars. (This last requirement is from the Norwegian concrete design code, as EC2 does not contain any particular requirements for such reinforcement.) It is the opinion of this structural engineer that it may be reduced according to the stress in the anchorage reinforcement. The cross bars shall consist of at least two bars placed inside the bend. The cross bars can be anchored with 90° hook at the ends or by being shaped like a U-bar.

Date :	18.10.05	Sign: tb
Last rev:	26.09.11	Sign: tb
Doc. No:	K4-10/39bE	Sign: tb
Page 6 of 12		

Reinforcement design for BCC 450

2.2.3 Minimum beam depth

This means, with dimensions as assumed in figure 2, without cross bars the minimum beam depth is $25+30+270+5+284/2+35+15+25 \approx 550$ mm

With minimum mandrel diameter the minimum beam depth is $25+30+270+5+224/2+35+15+25 = 515$ mm

If cross bars are required: $0,4 \cdot 804 \cdot 2 \cdot 355 / 435 = 525$ mm², e.g. 3-Ø12 U-shaped stirrups inserted from each side.

2.2.4 Anchorage

See figure 2. (The main reinforcement in the beam is not shown in the side view.)

According to EC2 clause 8.4.4:

$$l_{bd} = \alpha_1 \cdot \alpha_2 \cdot \alpha_3 \cdot \alpha_4 \cdot \alpha_5 \cdot l_{b,reqd} \geq l_{b,min}$$

$$l_{b,reqd} = \frac{\sigma}{4} \cdot \frac{\sigma_{sd}}{f_{bd}} = \frac{32}{4} \cdot \frac{355}{4,05} = 701 \text{ mm}$$

$$l_{b,min} = \max(0,3 \cdot l_{b,reqd}; 10 \cdot \emptyset; 100 \text{ mm}) = 320 \text{ mm}$$

Straight bar: $\alpha_1 = 1,0$

Concrete cover: $c_d = \min(a/2; c_1) = \min[(140-32)/2; (300-140-32)/2] = 54$ mm

$$\alpha_2 = 1 - 0,15 \cdot (c_d - 3 \cdot \emptyset) / \emptyset = 1 - 0,15 \cdot (54 - 3 \cdot 32) / 32 > 1,0 ; \text{i.e. } \alpha_2 = 1,0$$

Confinement by reinforcement: Assume transverse reinforcement Ø12-c/c150 mm:

$$\alpha_3 = 1 - K \cdot \lambda = 1 - 0,05 \cdot \frac{113 \cdot \frac{600}{150} - 0,25 \cdot 113}{113} = 0,813$$

Confinement by welded transverse reinforcement:

$$\alpha_4 = 1,0$$

Confinement by transverse pressure:

$$\alpha_5 = 1,0$$

$$l_{bd} = 1,0 \cdot 1,0 \cdot 0,813 \cdot 1,0 \cdot 1,0 \cdot 701 = 570 \text{ mm. } 800 \text{ mm recommended.}$$

Date :	18.10.05	Sign: tb
Last rev:	26.09.11	Sign: tb
Doc. No:	K4-10/39bE	Sign: tb
Page 7 of 12		

Reinforcement design for BCC 450

It must always be checked that the beam's main reinforcement has sufficient anchorage at the end of the horizontal part of the front anchorage. This will probably lead to greater lengths for the horizontal part of the front anchorage than calculated here.

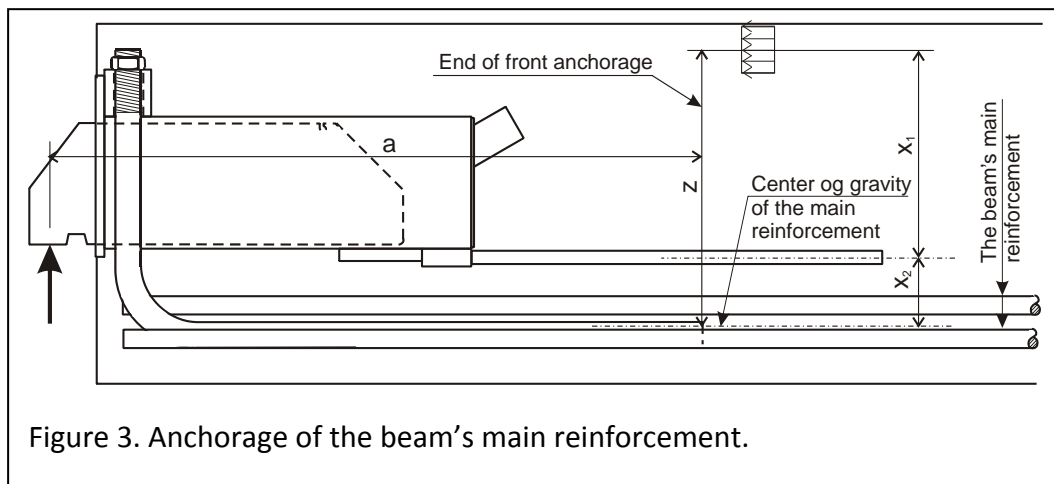
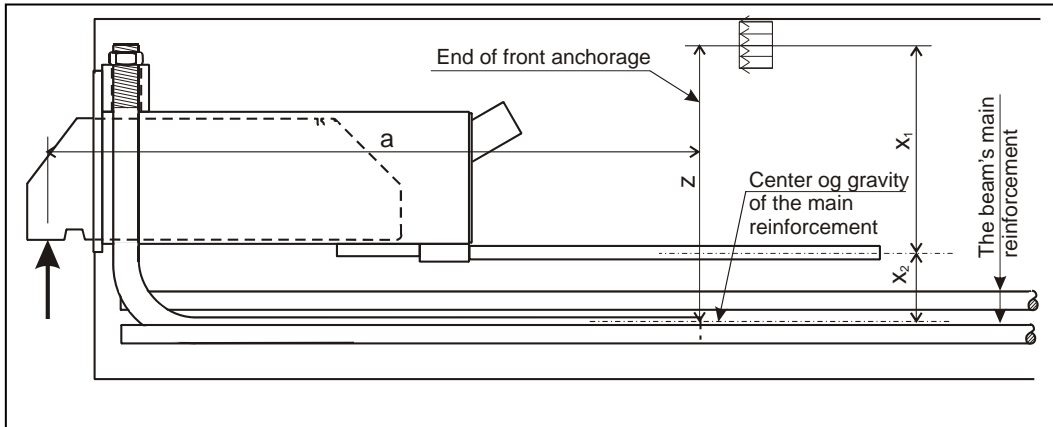


Figure 3. Anchorage of the beam's main reinforcement.

The force is approximately $[F_V \times \frac{a}{z} + \frac{F_V}{2} + F_H \frac{x_1}{z}]$. Additional force in the beam's top reinforcement is about $[F_H \cdot \frac{x_2}{z}]$, but this force leads to a reduction in the compressive stresses and may be neglected. If there is a large distance between the horizontal part of the front anchorage and the beam's main reinforcement, the length of the horizontal part of the front anchorage must be increased with this distance. (This distance is approximately zero in the figure.)

Date :	18.10.05	Sign: tb
Last rev:	26.09.11	Sign: tb
Doc. No:	K4-10/39bE	Sign: tb
Page 8 of 12		

Reinforcement design for BCC 450

2.2.5 Examples of detailing of the front anchorage

With dimensions as given in figure 2 and 430 mm mandrel diameter:

Minimum length of anchorage bar = $30+270+5+\pi \times 430/4+511 = 1154$ mm

The length of threads must be $30+15 = 45$ mm

If a length of 1200 mm is selected the length of threads should be $45+(1200-1154) = 90$ mm, or the front anchorage can be lowered 50 mm down in the beam – if there is space. That would be beneficial with regard to the anchorage capacity.

With dimensions as given in figure 2 and 220 mm mandrel diameter:

Minimum length of anchorage bar = $30+270+5+\pi \times 220/4+511 = 989$ mm

The length of threads must still be $30+15 = 45$ mm

If a length of 1200 mm is selected the length of threads should be $45+(1200-989) = 255$ mm, or the front anchorage can be lowered 200 mm down in the beam – if it is room. That would be beneficial with regard to the anchorage capacity.

2.3 BEAM UNIT – ANCHORAGE OF HORIZONTAL FORCES

2.3.1 Check of capacity

$F_H = 135$ kN (see clause 1.3)

$A_{s,reqd} = 135/0,512 = 264$ mm² (see clause 1.4)

M20 threaded bar (245 mm²) is assumed – 7 % too little – ok

(The stress in the bar will be $135 \cdot 10^3 / 245 = 551$ MPa. Different codes allow from 512 to 582 MPa in threaded bars of quality 8.8 in ULS.)

Date :	18.10.05	Sign: tb
Last rev:	26.09.11	Sign: tb
Doc. No:	K4-10/39bE	Sign: tb
Page 9 of 12		

Reinforcement design for BCC 450

2.3.2 Anchorage length

$$l_{bd} = \alpha_1 \cdot \alpha_2 \cdot \alpha_3 \cdot \alpha_4 \cdot \alpha_5 \cdot l_{b,reqd} \geq l_{b,min}$$

No information exists on the bond characteristics of threaded bars. Assume η_1 to be 0,7.

$$f_{bd} = 2,25 \cdot 0,7 \cdot 1,0 \cdot 1,80 = 2,84 \text{ MPa}; \quad l_{b,reqd} = \frac{\sigma_{sd}}{f_{bd}} = \frac{20}{2,84} = 7,05 \text{ mm}$$

$$l_{b,min} = \max(0,3 \cdot l_{b,reqd}; 10 \cdot \emptyset; 100 \text{ mm}) = 291 \text{ mm}$$

Straight bar: $\alpha_1 = 1,0$

Concrete cover: $c_d = \min(a/2; c_1; c) = \min(140/2; 100; 100) = 70 \text{ mm}$

$$\alpha_2 = 1 - 0,15 \cdot (c_d - 3 \cdot \emptyset) / \emptyset = 1 - 0,15 \cdot (70 - 3 \cdot 20) / 20 = 0,925$$

Confinement by reinforcement: Assume transverse reinforcement $\emptyset 12$ -c/c150 mm:

$$\alpha_3 = 1 - K \cdot \lambda = 1 - 0,05 \cdot \frac{113 \cdot \frac{600}{150} - 0,25 \cdot 113}{113} = 0,813$$

Confinement by welded transverse reinforcement: $\alpha_4 = 1,0$

Confinement by transverse pressure: $\alpha_5 = 1,0$

$$l_{bd} = 1,0 \cdot 0,925 \cdot 0,813 \cdot 1,0 \cdot 1,0 \cdot 970 = 730 \text{ mm}$$

Select 800 mm

Reinforcement design for BCC 450

2.4 COLUMN UNIT – ANCHORAGE IN THE COLUMN

2.4.1 Check of capacity

$$M = F_H \times (25 + 30/2) + F_V \times [(80/2) - 35] =$$

$$= 450 \times (0,3 \times 40 + 5) = 7650 \text{ kNmm}$$

$$S = 7650 / 289 = 26,5 \text{ kN}$$

$$A_s = 26,5 / 0,512 = 52 \text{ mm}^2$$

M10 at the top is ok (see table in clause 1.4).

$$F_H - S = 0,3 \times 450 - 26,5 = 108,5 \text{ kN}$$

$$A_s = 108,5 / 0,512 = 212 \text{ mm}^2$$

M20 in the bottom is ok

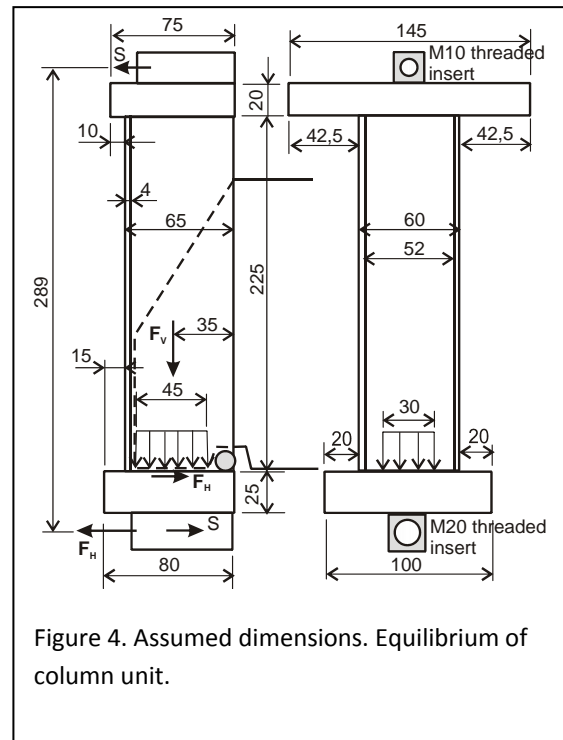


Figure 4. Assumed dimensions. Equilibrium of column unit.

2.4.2 Anchorage of the bolts

See separate calculations: "Design of anchorages used in the BCC units."

See also Memo 27.

Date :	18.10.05	Sign: tb
Last rev:	26.09.11	Sign: tb
Doc. No:	K4-10/39bE	Sign: tb
Page 11 of 12		

Reinforcement design for BCC 450

PART 3 – REINFORCEMENT

3.1 BEAM – REINFORCEMENT IN THE REAR

$$A_{s,reqd} = R_{CU}/f_{sd} = 122/0,435 = 280 \text{ mm}^2,$$

corresponding to $280/(2 \times 113) = 1,24$; i.e. 2 – Ø12 stirrups.

3.2 BEAM – SHEAR STIRRUPS

Use a strut-and-tie model with compressive diagonals at 45°.

The shear force within the central part of the beam unit is assumed to be $R_{CO} = 572 \text{ kN}$.

Assume $z = 500 \text{ mm}$ and stirrup diameter Ø12.

$$s = A_{sw} \cdot z \cdot f_{yd} \cdot \cot \theta / V$$

$$s = 2 \times 113 \times 435 \times 500 \times \cot 45^\circ / 572 \times 10^3 = 86 \text{ mm}$$

Select Ø12 c/c 80 mm. This reinforcement should be brought approximately 200 mm past the end of the beam unit in order to absorb any splitting effects from the anchorage of the threaded bar.

3.3 BEAM – CHECK OF SHEAR COMPRESSION

Assume beam width of 300 mm and beam depth of 550 mm. (See also clause 2.2.) This means that $z_{min} \approx 550 - 50 - 100 = 400 \text{ mm}$ with one layer of reinforcement in the top of the beam and three layers in the bottom.

The shear reinforcement is calculated based on 45° compressive field. The check of shear compression will be carried out under the same assumption:

$$V_{Rd,max} = \alpha_{cw} \cdot b_w \cdot z \cdot v_1 \cdot f_{cd} / (\cot \theta + \tan \theta)$$

$$V_{Rd,max} = \{1,0 \cdot (300 - 100) \cdot 400 \cdot 0,6 \cdot [1 - (45/250)] \cdot 30,0 / (\cot 45^\circ + \tan 45^\circ)\} \cdot 10^{-3}$$

$$V_{Rd,max} = 1181 \text{ MPa} > 570 \text{ MPa} - \text{OK}$$

3.4 COLUMN – SPLITTING STRESS

Splitting stress under the column unit according to Leonhardt $\approx 0,2 \times F_V = 0,2 \times 450 = 90 \text{ kN}$

$$A_S = 90/0,435 = 207 \text{ mm}^2$$

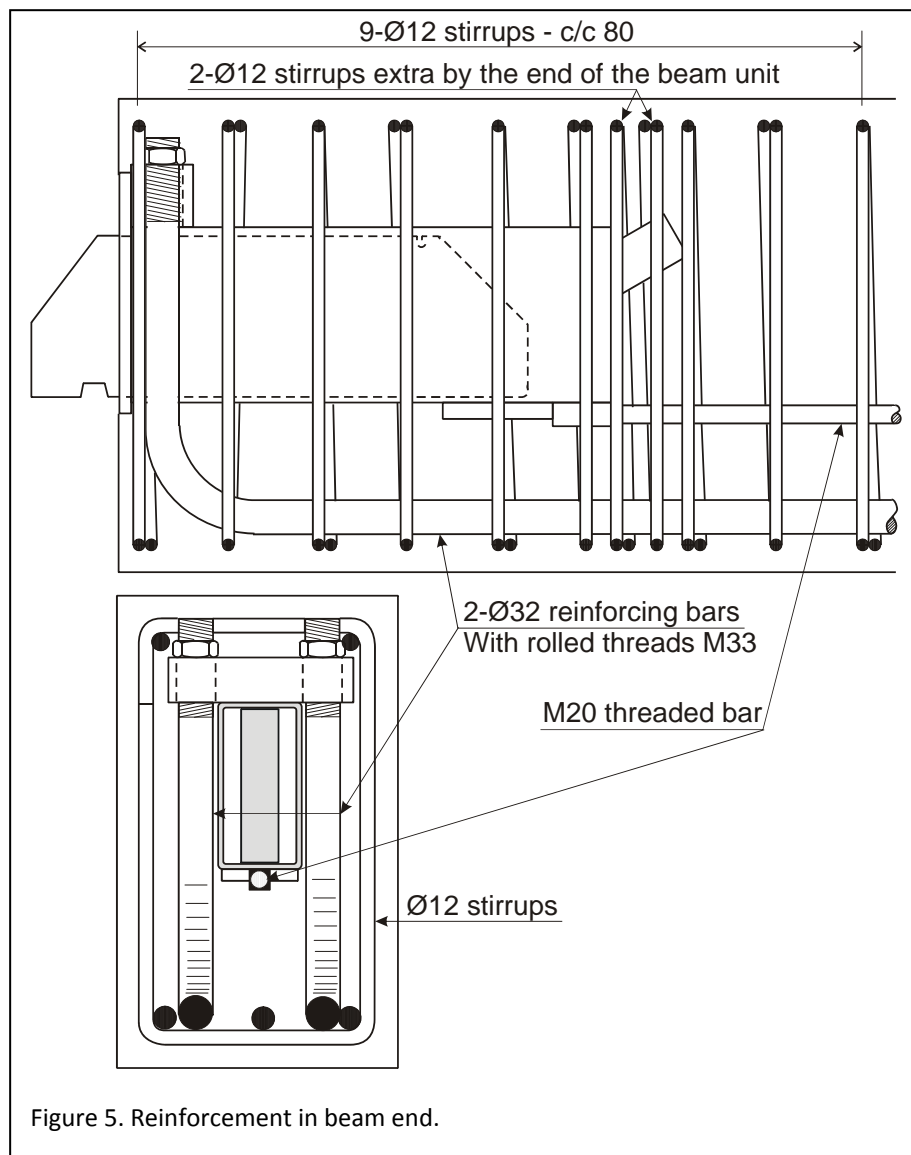
1-Ø12 stirrup ($2 \times 113 \text{ mm}^2$) is sufficient.

Date :	18.10.05	Sign: tb
Last rev:	26.09.11	Sign: tb
Doc. No:	K4-10/39bE	Sign: tb
Page 12 of 12		

Reinforcement design for BCC 450

3.5 CONCLUSION – REINFORCEMENT IN THE BEAM

For clarity the beam's main reinforcement is not shown in the side view. Between the shown stirrups in each end of the beam a normal calculation of the shear reinforcement must be carried out. The main reinforcement in the beam must of course also be calculated.



At the end of the front anchorage it must be checked that the beam's main reinforcement has sufficient anchorage. See clause 2.2.4.