

Memo 39c

Date :	18.10.05	Sign: tb	
Last rev:	26.09.11	Sign: tb	
Doc. No:	K4-10/39cE	Sign: tb	
Page 1 of 12			

Reinforcement design for BCC 800

PART 1 – BASIC ASSUMPTIONS

1.1 GENERAL

In these calculations certain assumptions has been made about dimensions and qualities in the precast concrete elements that may not always be the case. Therefore the following calculations of anchorage of the units and the resulting reinforcement must be considered as an example to illustrate the calculation model.

In beams it must always be checked that the forces from the anchorage reinforcement can be transferred to the beam's main reinforcement. The recommended shear reinforcement (stirrups) includes all necessary stirrups in the beam end; i.e. the normal shear reinforcement in beam ends and an addition due to the cantilever action of the beam unit from the beam.

The information found here and in the memos assumes that the design of the elements and the use of the units in structural elements are carried out under the supervision of a structural engineer with profound knowledge of the behaviour of concrete structures.

1.2 STANDARDS

The calculations are carried out according to:

Eurocode 2: Design of concrete structures. Part 1-1. General rules and rules for buildings.

Eurocode 3: Design of steel structures. Part 1-1: General rules and rules for buildings.

Eurocode 3: Design of steel structures. Part 1-8: Design of joints.

EN 10080: Steel for the reinforcement of concrete. Weldable reinforcing steel. General.

For all NPDs in the Eurocodes the recommended values are used.

These NDPs are: In EC 2: γ_c ; γ_s ; α_{cc} ; α_{ct} ; k_1 ; k_2 and $\emptyset_{m,min}$. In EC 3: γ_{M0} ; γ_{M1} and γ_{M2} .

1.3 LOADS

Vertical ultimate limit state load = $F_V = 800 \text{ kN}$

Horizontal ultimate limit state load = $F_H = 0.3 \times F_V = 240 \text{ kN}$

The design is carried out with a horizontal force of 30 % of the vertical force. Recommended capacity for transfer of active forces (as wind loads) may be assumed to be 20 % of smallest vertical load present at the same time.



Memo 39c

Date :	18.10.05	Sign: tb
Last rev:	26.09.11	Sign: tb
Doc. No:	K4-10/39cE	Sign: tb
Page 2 of 12		

Reinforcement design for BCC 800

1.4 QUALITIES

Concrete grade C45/55: $f_{cd} = \alpha_{cc} \cdot f_{ck} / \gamma_c = 1,0.45/1,5 = 30,0 \text{ MPa}$

 $f_{ctd} = \alpha_{ct} \cdot f_{ctk,0,05} / \gamma_c = 1,0 \cdot 2,70/1,5 = 1,80 \text{ MPa}$

 $f_{bd} = 2,25 \cdot \eta_1 \cdot \eta_2 \cdot f_{ctd} = 2,25 \cdot 1,0 \cdot 1,0 \cdot 1,80 = 4,05 \text{ MPa}$

Reinforcement B500C: $f_{yd} = f_{yk}/\gamma_s = 500/1,15 = 435 \text{ MPa}$

Tension in threaded bars: 8.8 quality steel: $f_{yd} = f_y/\gamma_{M2} = 640/1,25 = 512$ MPa

Nominal diameter (mm)	M10	M12	M16	M20	M24	M30	M33	M36
Equivalent diameter (mm)	8,6	10,4	14,1	17,7	21,2	26,7	29,7	32,6
Stress area (mm ²)	58	84	157	245	353	561	694	835

Structural steel S355:

Tension and compression:

$$f_{yd} = f_y / \gamma_{M0} = 355/1,00 = 355 \text{ MPa} < f_u / \gamma_{M2} = 510/1,25 = 408 \text{ MPa}$$

Shear: $f_{sd} = f_v/(\gamma_{M0} \cdot \sqrt{3}) = 355/(1,00 \cdot \sqrt{3}) = 205 \text{ MPa}$

Welds:
$$f_{vw.d} = \frac{f_u}{\gamma_{W.2}\sqrt{3}} \times \frac{1}{\beta_w} = \frac{510}{1.25 \times \sqrt{3}} \times \frac{1}{0.9} = 262 \text{ MPa}$$



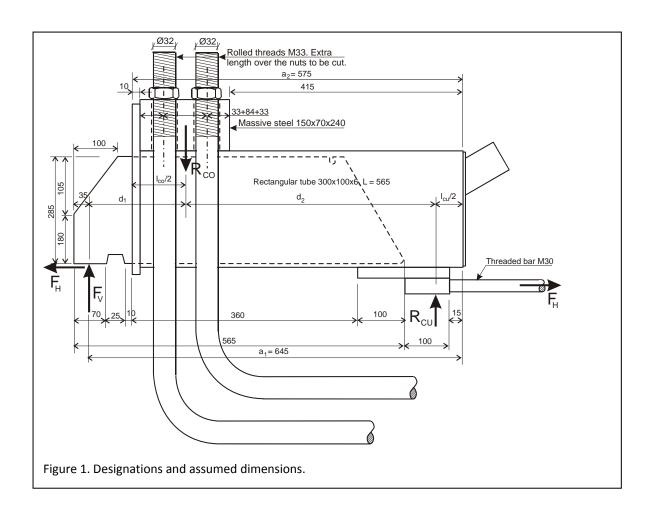
Memo 39c

Date :	18.10.05	Sign: tb		
Last rev:	26.09.11	Sign: tb		
Doc. No:	K4-10/39cE	Sign: tb		
Page 3 of 12				

Reinforcement design for BCC 800

PART 2 – ANCHORAGE OF THE UNITS

2.1 BEAM UNIT – EQUILIBRIUM



Neglecting the moment created by the small vertical shift in the transfer of F_H.

$$R_{CU} = F_V \times d_1/d_2$$

$$R_{\rm CO} = F_{\rm V} + R_{\rm CU}$$

 $l_{CU} = R_{CU} / (f_{cd} \times b)$ (b is the width of the beam unit.)

 $l_{CO}/2$ is given geometrically:

$$l_{CO}/2 = 10 + 33 + 84/2 = 85 \text{ mm}$$

$$d_1 = a_1 - a_2 + l_{CO}/2$$

$$d_2 = a_2 - l_{CO}/2 - l_{CU}/2$$



Memo 39c

Date :	18.10.05	Sign: tb
Last rev:	26.09.11	Sign: tb
Doc. No:	K4-10/39cE	Sign: tb
Page 4 of 12		

Reinforcement design for BCC 800

Use a spread sheet. Assume d_1/d_2 , calculate resulting d_1/d_2 . Change assumed value until they are equal. (Spread sheet "Equilibrium-BCC800-beam-unit.exc".)

	Equili	brium of BCC 800 beam unit:		
		Concrete grade = C	45	/55
Geometry:		Material coefficient for concrete =	1,5	
$a_1 =$	645	mm $f_{cd} =$	30,0	MPa
$a_2 =$	575	mm		
Width of beam unit =	100	mm $F_V =$	800	kN
$l_{CO}/2 =$	85	mm		
		Assumed $d_1 / d_2 = 0.349579891$		
R _{CU} =	280	kN $d_1 =$	155	mm
$R_{CO} =$	1080	kN $d_2 =$	443	mm
$l_{CU} =$	93	mm		
		Calculated $d_1/d_2 = 0.349579891$		



Memo 39c

Date :	18.10.05	Sign: tb	
Last rev:	26.09.11	Sign: tb	
Doc. No:	K4-10/39cE	Sign: tb	
Page 5 of 12			

Reinforcement design for BCC 800

2.2 BEAM UNIT – ANCHORAGE IN FRONT

2.2.1 Check of capacity

$$A_{s,regd} = R_{CO}/f_{vd,reinf} = 1080/0,435 = 2483 \text{ mm}^2$$

Assumed
$$4-@32 = 4\times804 = 3216 \text{ mm}^2 - \text{ok}$$

2.2.2 Bending of front reinforcement

See figure 2. Stress in anchorage reinforcement = $1080 \times 10^3 / 3216 = 335 \text{ MPa}$

According to EN 1991-1-1, clause 8.3:

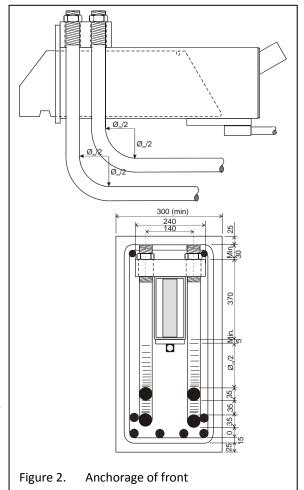
 $Q_{m,min}$ is generally 7.32 = 224 mm to avoid damage to the reinforcing bars.

To avoid cross bars:

$$\emptyset_{m,\min} = \frac{F_{bt} \cdot (\frac{1}{a_b} + \frac{1}{2} \cdot \emptyset)}{f_{cd}}$$

$$\emptyset_{m,\min} = \frac{1080 \cdot \frac{804}{3216} \cdot 10^3}{30} \cdot (\frac{1}{140/2} + \frac{1}{2 \cdot 32}) = 269mm$$

For mandrel diameters equal to or larger than 269 mm there is no requirement to transverse reinforcement in



the bend. Small distance from the side of the beam to the anchorage bars may still make cross bars necessary.

With concrete grade C45/55 transverse reinforcement in the bend will be required for

$$224 \text{ mm} \le \emptyset_m \le 270 \text{ mm}$$

For other concrete grades and bar diameters the mandrel diameters will be different.

If cross bars are required the total area of this reinforcement shall be at least 40% of the area of the bent bars. (This last requirement is from the Norwegian concrete design code, as EC does not contain any particular requirements for such reinforcement.) It is the opinion of this structural engineer that it may be reduced according to the stress in the anchorage reinforcement. The cross bars shall consist of at least two bars placed inside the bend. The cross bars can be anchored with 90° hook at the ends or by being shaped like a U-bar.



Memo 39c

Date :	18.10.05	Sign: tb	
Last rev:	26.09.11	Sign: tb	
Doc. No:	K4-10/39cE	Sign: tb	
Page 6 of 12			

Reinforcement design for BCC 800

2.2.3 Minimum beam depth

This means, with dimensions as assumed in figure 2, without cross bars the minimum depth is $25+30+370+5+270/2+3\times35+15+25\approx725$ mm

With minimum mandrel diameter the minimum depth is $25+30+370+5+224/2+3\times35+15+25\approx700$ mm

However, with the horizontal part of the front anchorage so close to the bottom of the beam unit, the unit and the bars may be torn out as one piece. Therefore a minimum beam depth of 800 mm is recommended.

If cross bars are required: $0.4 \cdot 804 \cdot 4 \cdot 335/435 = 991 \text{ mm}^2$

If U-shaped bars are used they will be properly anchored only in one end. Hence five U-shaped bars Ø12 inserted from each side are required.

(Total area =
$$2.2.5.113 = 2260 \text{ mm}^2 > 2.991 = 1982 \text{ mm}^2$$
.)

2.2.4 Anchorage

See figure 2. (The main reinforcement in the beam is not shown in the side view.) According to EC2 clause 8.4.4:

$$l_{bd} = \alpha_1 \cdot \alpha_2 \cdot \alpha_3 \cdot \alpha_4 \cdot \alpha_5 \cdot l_{b,reqd} \ge l_{b,min}$$

$$l_{b,reqd} = \frac{\emptyset}{4} \cdot \frac{\sigma_{sd}}{f_{bd}} = \frac{32}{4} \cdot \frac{335}{4,05} = 662 \text{ mm}$$

 $l_{b,min} = max(0,3 \cdot l_{b,reqd}; 10 \cdot \emptyset; 100 \text{ mm}) = 320 \text{ mm}$

Straight bar: $\alpha_1 = 1.0$

Concrete cover: $c_d = min(a/2; c_1) = min[(140-32)/2; (300-140-32)/2] = 54 \text{ mm}$

$$\alpha_2 = 1 - 0.15 \cdot (c_d - 3 \cdot \emptyset) / \emptyset = 1 - 0.15 \cdot (54 - 3 \cdot 32) / 32 > 1.0$$
; i.e. $\alpha_2 = 1.0$

Confinement by reinforcement: Assume transverse reinforcement Ø12-c/c150 mm:

$$\alpha_3 = 1 - K \cdot \lambda = 1 - 0.05 \cdot \frac{113 \cdot \frac{600}{150} - 0.25 \cdot 113}{113} = 0.813$$

Confinement by welded transverse reinforcement:

$$\alpha_4 = 1.0$$

Confinement by transverse pressure:

$$\alpha_5 = 1.0$$
 $l_{bd} = 1.0 \cdot 1.0 \cdot 0.813 \cdot 1.0 \cdot 1.0 \cdot 662 = 538 \text{ mm}$

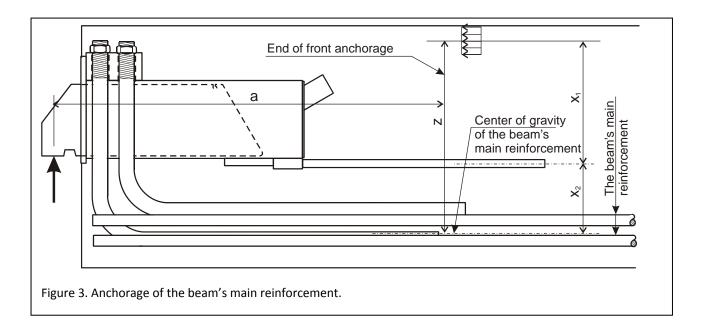


Memo 39c

Date :	18.10.05	Sign: tb		
Last rev:	26.09.11	Sign: tb		
Doc. No:	K4-10/39cE	Sign: tb		
Page 7 of 12				

Reinforcement design for BCC 800

It must always be checked that the beam's main reinforcement has sufficient anchorage at the end of the horizontal part of the front anchorage. This will probably lead to greater lengths for the horizontal part of the front anchorage than calculated here.



The force is approximately $[F_V \times \frac{a}{z} + \frac{F_V}{2} + F_H \frac{x_1}{z}]$. Additional force in the beam's top reinforcement is about

 $[F_H \cdot \frac{x_2}{z}]$, but this force leads to a reduction in the compressive stresses and may be neglected. If there is a

large distance between the horizontal part of the front anchorage and the beam's main reinforcement, the length of the horizontal part of the front anchorage must be increased with this distance. (This distance is approximately zero in the figure.)



Memo 39c

Date :	18.10.05	Sign: tb	
Last rev:	26.09.11	Sign: tb	
Doc. No:	K4-10/39cE	Sign: tb	
Page 8 of 12			

Reinforcement design for BCC 800

2.2.5 Examples of detailing of the front anchorage

With dimensions as given in figure 2 and with 820 mm mandrel diameter:

Minimum length of anchorage bars with the lower horizontal part:

$$1 = 30 + 370 + 5 + 2 \cdot 35 + \pi \cdot 820/4 + 483 = 1567 \text{ mm}$$

Minimum length of anchorage bars with the upper horizontal part:

$$1 = 30 + 370 + 5 + \pi \times 820/4 + 483 = 1532 \text{ mm}$$

For both of these the length of threads must be 30+15 = 45 mm

If a length of 1713 mm is selected (seven bars from a 12 m length) the length of threads should be 45+(1713-1567) = 191 mm, or the front anchorage can be lowered 100 mm down in the beam – it there is space. That would be beneficial with regard to the anchorage capacity.

With dimensions as given in figure 2 and 220 mm mandrel diameter:

Minimum length of anchorage bars with the lower horizontal part:

$$1 = 30 + 370 + 5 + 2 \cdot 35 + \pi \cdot 220/4 + 483 = 1096 \text{ mm}$$

Minimum length of anchorage bars with the upper horizontal part:

$$1 = 30 + 370 + 5 + \pi \times 220/4 + 483 = 1061 \text{ mm}$$

For both of these the length of threads must still be 30+15 = 45 mm

If a length of 1200 mm is selected (ten bars from a 12 m length) the length of threads should be 45+(1200-1096) = 149 mm, or the front anchorage can be lowered 100 mm down in the beam – it there is space. That would be beneficial with regard to the anchorage capacity.

1000 mm length may be selected (12 bars from a 12m length), which a little less than required. This can be accepted because it is assumed no anchorage (bond stresses) along the bent part of the bar.



Memo 39c

Date :	18.10.05	Sign: tb
Last rev:	26.09.11	Sign: tb
Doc. No:	K4-10/39cE	Sign: tb
Page 9 of 12		

Reinforcement design for BCC 800

2.3 BEAM UNIT – ANCHORAGE OF HORIZONTAL FORCES

2.3.1 Check of capacity

 $F_H = 240 \text{ kN}$ (see clause 1.3)

 $A_{s,reqd} = 240/0,512 = 469 \text{ mm}^2 \text{ (See clause 1.4)}$

One M30 threaded bar is assumed – ok (see clause 1.4)

The stress in the bar is 240/0,561 = 428 MPa

2.3.2 Anchorage length

$$l_{bd} = \alpha_1 \cdot \alpha_2 \cdot \alpha_3 \cdot \alpha_4 \cdot \alpha_5 \cdot l_{b,reqd} \ge l_{b,min}$$

No information exists on the bond characteristics of threaded bars. Assume η_1 to be 0,7.

$$f_{bd} = 2,25 \cdot 0,7 \cdot 1,0 \cdot 1,80 = 2,84 \text{ MPa}; \ l_{b,reqd} = \frac{\emptyset}{4} \cdot \frac{\sigma_{sd}}{f_{bd}} = \frac{30}{4} \cdot \frac{428}{2,84} = 1130 \text{ mm}$$

 $l_{b,min} = max(0,3 \cdot l_{b,reqd}; 10 \cdot \emptyset; 100 \text{ mm}) = 339 \text{ mm}$

Straight bar: $\alpha_1 = 1.0$

Concrete cover: $c_d = min(a/2; c_1; c) = min(140/2; 100; 100) = 70 \text{ mm}$

$$\alpha_2 = 1 - 0.15 \cdot (c_d - 3 \cdot \emptyset) / \emptyset = 1 - 0.15 \cdot (70 - 3 \cdot 30) / 30 > 1.0$$
; i.e. $\alpha_2 = 1.0$

Confinement by reinforcement: Assume transverse reinforcement Ø12-c/c150 mm:

$$\alpha_3 = 1 - K \cdot \lambda = 1 - 0.05 \cdot \frac{113 \cdot \frac{600}{150} - 0.25 \cdot 113}{113} = 0.813$$

Confinement by welded transverse reinforcement: $\alpha_4 = 1.0$

Confinement by transverse pressure: $\alpha_5 = 1.0$

$$l_{bd} = 1,0.1,0.0,813.1,0.1,0.1130 = 919 \text{ mm}$$

Select 923 mm (13 bars from a 12 m length).



Memo 39c

Date :	18.10.05	Sign: tb
Last rev:	26.09.11	Sign: tb
Doc. No:	K4-10/39cE	Sign: tb
Page 10 of 12		

Reinforcement design for BCC 800

2.4 COLUMN UNIT – ANCHORAGE IN THE COLUMN

2.4.1 Check of capacity

$$M = F_H \times (30+32/2) + F_V \times [(120/2)-55] =$$

$$= 800 \times (0.3 \times 46+5) = 15040 \text{ kNmm}$$

$$S = 15040/400 = 37,6 \text{ kN}$$

$$A_s = 37,6/0,512 = 73 \text{ mm}^2$$

M12 at the top is ok (see table in clause 1.4).

$$F_H - S = 0.3 \times 800 - 38 = 202 \text{ kN}$$

$$A_s = 202/0.512 = 395 \text{ mm}^2$$

M24 threaded bar (353 mm²) is assumed –

10 % too little – ok

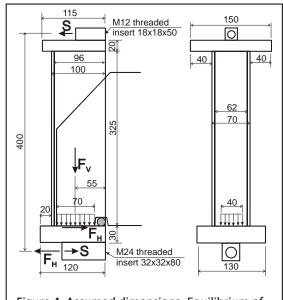


Figure 4. Assumed dimensions. Equilibrium of column unit.

(The stress in the bar will be $202 \cdot 10^3 / 353 = 572$ MPa. Different codes allow from 512 to 582 MPa in threaded bars of quality 8.8 in ULS.)

2.4.2 Anchorage of the bolts

See separate calculations: "Design of anchorages used in the BCC units."

See also Memo 27.



Memo 39c

Date :	18.10.05	Sign: tb
Last rev:	26.09.11	Sign: tb
Doc. No:	K4-10/39cE	Sign: tb
Page 11 of 12		

Reinforcement design for BCC 800

PART 3 – REINFORCEMENT

3.1 BEAM – REINFORCEMENT IN THE REAR

$$A_{s,regd} = R_{CU}/f_{sd} = 280/0,435 = 644 \text{ mm}^2,$$

Corresponding to $644/(2\times201) = 1.6$; i.e. $2 - \emptyset16$ stirrups.

(With three Ø12 stirrups the stress in the reinforcement will be

$$280 \times 10^{3} / 2 \times 3 \times 113 = 413$$
 MPa, which also is ok.)

3.2 BEAM – SHEAR STIRRUPS

See EC2, clause 6.2.3 (3).

Use a strut-and-tie model with compressive diagonals at 45°.

The shear force within the central part of the beam unit is assumed to be $R_{CO} = 1080 \text{ kN}$.

Assume z = 700 mm and stirrup diameter Ø12.

$$s = s = A_{sw} \cdot z \cdot f_{vd} \cdot \cot \theta / V$$

$$s = 2 \times 113 \times 435 \times 700 \times cot \ 45^{\circ} / \ 1080 \times 10^{3} = 64 \ mm$$

Select Ø12 c/c 60 mm. This reinforcement should be brought approximately 300 mm past the end of the beam unit in order to absorb any splitting effects from the anchorage of the threaded bar.

3.3 BEAM - CHECK OF SHEAR COMPRESSION

Assume beam width of 300 mm and beam depth of 800 mm. (See also clause 2.2.) This means that $z_{min} \approx 800-50-100 = 650$ mm with one layer of reinforcement in the top of the beam and three layers in the bottom.

The shear reinforcement is calculated based on 45° compressive field. The check of shear compression will be carried out under the same assumption:

$$V_{Rd.max} = \alpha_{cw} \cdot b_w \cdot z \cdot v_1 \cdot f_{cd} / (\cot \theta + \tan \theta)$$

$$V_{Rd,max} = \{1,0 \cdot (300-100) \cdot 650 \cdot 0,6 \cdot [1-(45/250)] \cdot 30,0/(\cot 45^{\circ} + \tan 45^{\circ})\} \cdot 10^{-3}$$

$$V_{Rd,max} = 1181 \text{ MPa} > 1080 \text{ MPa} - \text{OK}$$

3.4 COLUMN – SPLITTING STRESS

Splitting stress under the column unit according to Leonhardt $\approx 0.2 \times F_V = 0.2 \times 800 = 160 \text{ kN}$

$$A_S = 160/0,435 = 368 \text{ mm}$$

2-Ø12 stirrups ($2\times2\times113 \text{ mm}^2$) is sufficient



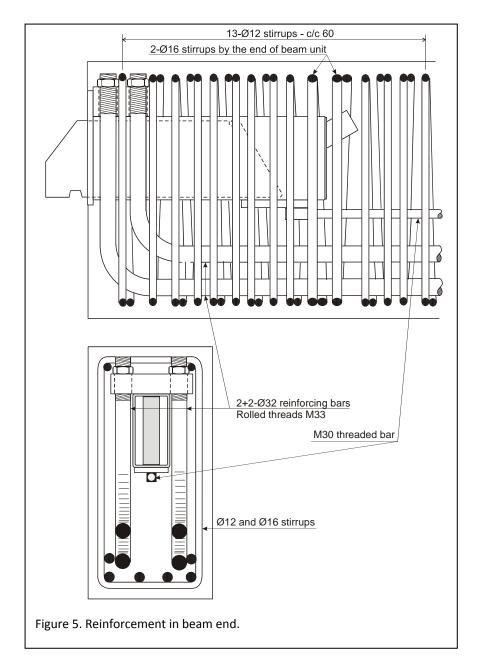
Memo 39c

Date :	18.10.05	Sign: tb
Last rev:	26.09.11	Sign: tb
Doc. No:	K4-10/39cE	Sign: tb
Page 12 of 12		

Reinforcement design for BCC 800

3.5 CONCLUSION – REINFORCEMENT IN THE BEAM

For clarity the beam's main reinforcement is not shown in the side view. Between the shown stirrups in each end of the beam a normal calculation of the shear reinforcement must be carried out. The main reinforcement in the beam must of course also be calculated.



At the end of the front anchorage it must be checked that the beam's main reinforcement has sufficient anchorage. See clause 2.2.4.