Design of reinforced concrete sections according to EN 1992-1-1 and EN 1992-2

Examples – part 2 - 2D elements

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Content

1. Verification examples

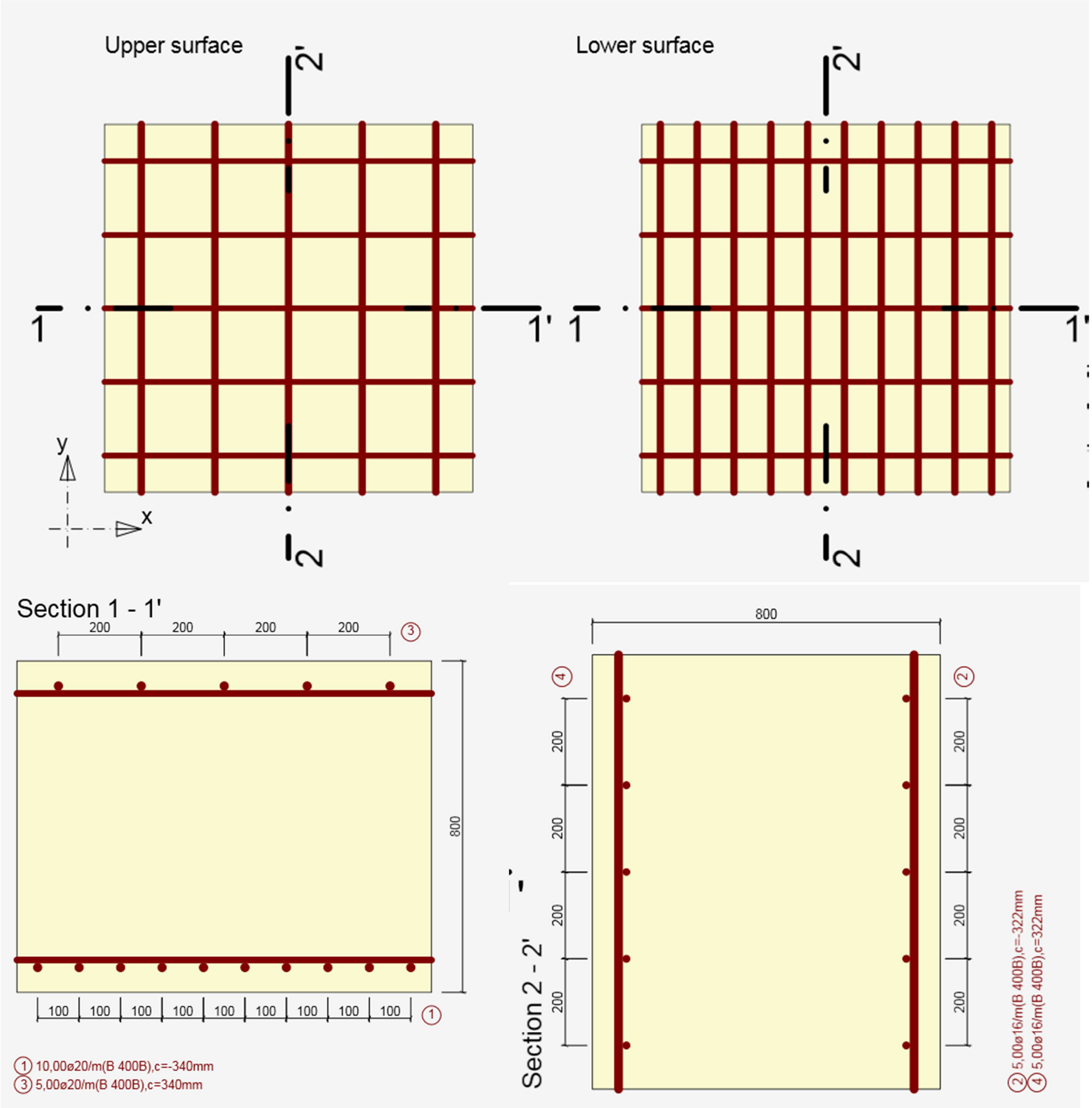
1.1. Comparison of capacity check and crack width check on a sandwich model according to ČSN EN 1992 – Appendix LL and in IDEA Concrete

The process of check for shell elements with or without cracks is described in appendix LL of [2], which are loaded by eight components of internal forces. The results of manual calculation are compared with results of IDEA Concrete. The input parameters are taken from [1].

1.1.1. Input values of cross-section and loads

For the verification example an element of base plate was chosen, the thickness is 0,8 m, reinforced at both surfaces with reinforcement B 400B (the equivalent reinforcement V10425 is used in manual calculation). In the x-direction at the lower surface the reinforcement ϕ16 with distances 200 mm (A_{sx} = 1005 mm^2) is designed, at the upper surface there is the same reinforcement ϕ16 with distances 200 mm. In the transversal y-direction at the lower surface the reinforcement ϕ20 with distances 100 mm (A_{sy} = 3142 mm^2) is designed, at the upper surface reinforcement ϕ20 with distances 200 mm (A_{sy} = 1571 mm^2). Concrete quality is C25/30.

The scheme of reinforced cross-section in IDEA Concrete



Reinforcement layers

Layer	Ø [mm]	n [/m]	Distance [mm]	Type first bar position	First bar position [mm]	Angle [°]	As [mm2]	Surface for cover	Position type	Value [mm]	Type
1	20	10,00	100	Distance / 2	50	90,0	3142	Lower	By clear cover	50	Main
2	16	5,00	200	Distance / 2	100	180,0	1005	Lower	By clear distance from layer 1	0	Main
3	20	5,00	200	Distance / 2	100	90,0	1571	Upper	By clear cover	50	Main
4	16	5,00	200	Distance / 2	100	180,0	1005	Upper	By clear distance from layer 3	0	Main

1.1.2. Loading of element

Loads entered in IDEA Concrete, values correspond to manual example:

Combination type	mx [kNm/m]	my [kNm/m]	mxy [kNm/m]	nx [kN/m]	ny [kN/m]	nxy [kN/m]	vx [kN/m]	vy [kN/m]
Fundamental ULS	171,40	1165,90	0,00	-1704,50	-1785,70	0,00	-0,90	39,40
Characteristic	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Quasi-permanent	171,40	1165,90	0,00	-1704,50	-1785,70	0,00	0,00	0,00

Sign convention for calculation:

- Positive moment causes tension in lower fibres
- Positive normal force causes tension in cross-section

1.1.3. Check of shear reinforcement

The check, if shear reinforcement is required, is proven by comparison of values $V_{Rd,c}$ and V_{Ed} according to formulas

$$V_{Rd,c} = \left[C_{Rd,c} \cdot k \cdot (100 \cdot \rho_1 \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp}\right] \cdot b_w \cdot d$$

$$v_{Edo} = \sqrt{v_{Edx}^2 + v_{Edy}^2}$$

Value	V_{Ed} [kN/m]	$V_{Rd,c}$ [kN/m]	Shear reinforcement
Manual calculation	39,4	471,0	No
IDEA Concrete	39,4	544,4	No

The difference of $V_{Rd,c}$ is caused by different inclusion of longitudinal reinforcement into ρ_l , which is not explained in the manual calculation. Value in IDEA Concrete corresponds to the area of tensioned reinforcement at the lower surface A_{sy} = 3142 mm^2. But it is not required to design shear reinforcement in both cases.

Table with results of interaction of shear force, bending moment and normal forces in IDEA Concrete, where both values are present:

Interaction

Angle between x-axis and checked direction : 90,0°

N Ed [kN]	M Edy [kNm]	V Ed [kN]	V Rd,c [kN]	V Rd,max [kN]	Value V+M [%]	Value [%]	Limit [%]	Check
-1785,70	1149,89	39,41	544,37	3508,63	88,02	88,02	100,00	OK

1.1.4. Check of cracking

For next calculation according to Appendix LL it has to be determined, if the cross-section is cracked or not by verifying the stress at different levels of the element thickness. The verification is calculated using following formula:

$$\Phi = \alpha \frac{J_2}{f_{cm}{}^2} + \lambda \frac{\sqrt{J_2}}{f_{cm}} + \beta \frac{I_1}{f_{cm}} - 1 \leq 0 \qquad \text{(LL.101)}$$

If the inequality is fulfilled, the element is considered as non-cracked.

The inequality is not fulfilled in manual calculation, thus the cross-section has to be considered as cracked in following calculations

$$\Phi = 3{,}896\frac{26{,}86}{33^2} + 14{,}572\frac{\sqrt{26{,}86}}{33} + 4{,}42\frac{-8{,}17}{33} - 1 = 0{,}09609 + 2{,}2885 - 1{,}094 - 1 = 0{,}291 \leq 0$$

IDEA Concrete calculates the value of crack width in the element depending on current element load and reinforcement. Results of crack width calculation are displayed in the following table:

Crack width

Angle between x-axis and checked direction : 90,0°

Crack width - short-term effect

N [kN]	M_y [kNm]	M_z [kNm]	w_k [mm]	w_{lim} [mm]	Value [%]	Limit [%]	Check
-1785,70	1149,89	0,00	0,330	0,400	82,61	100,00	OK

Intermediate results and coefficients for crack width calculation - short-term effect

x [mm]	$h_{c,eff}$ [mm]	d [mm]	$A_{c,eff}$ [mm2]	$A_{s,eff}$ [mm2]	$\rho_{p,eff}$ [-]
240	150	740	150000	3142	0,02
k_t [-]	$\varepsilon_{sm} - \varepsilon_{cm}$ [1e-4]	k_1 [-]	k_2 [-]	k_3 [-]	k_4 [-]
0,40	11,1	0,80	0,50	3,40	0,43
c [mm]	ε_1 [1e-4]	ε_2 [1e-4]	$s_{r,max}$ [mm]	Φ [mm]	σ_s [MPa]
40	15,5	-6,6	298	20	277,02

According to the values in the table the crack of width 0.330 mm appears in the y-axis direction (90 ^{o} to the x-axis), thus the cross-section is cracked too.

1.1.5. Dimensional forces

In the Appendix LL it is described, how to calculate axial forces in the outer layers of the sandwich model for check of the reinforcement in those layers.

IDEA Concrete calculates dimensional forces using the Baumann theory into the particular check directions at both element surfaces. Thus both methods can be simply compared in the following table.

Values represent normal forces in given direction and at given element surface.

Value	$n_{Edx,upper}$ [kN/m]	$n_{Edx,lower}$ [kN/m]	$n_{Edy,upper}$ [kN/m]	$n_{Edy,lower}$ [kN/m]
Manual calculation	-1104,3	-600,2	-2607,4	821,7
IDEA Concrete	-1111,3	-593,4	-2653,8	868,1

The table shows good compliance of the calculated forces. Only the tension in y-axis direction at the lower surface appears in both cases. In the other cases the stress is carried by concrete.

IDEA Concrete calculation results are displayed in the following table:

Recalculated design forces

Design forces in centroidal plane for ULS combination

Angle [°]	Concrete strut	n_{upper} [kN/m]	n_{lower} [kN/m]	n_d [kN/m]	m_d [kNm/m]	v_d [kN/m]
0,0	No	-1111,13	-593,37	-1704,50	156,12	39,41
90,0	No	-2653,83	868,13	-1785,70	1149,89	39,41

1.1.6. Check of capacity

It remains to check the capacity of reinforced cross-section. The total capacity is not calculated in the manual calculation, it is only to prove, that the real reinforcement is greater than the required design reinforcement and it is calculated on tension normal forces.

The required reinforcement area 2304 mm^2 is determined for the y-direction at the lower surface, which is less than the defined reinforcement of area 3142 mm^2. Dividing those values gives the exploitation of cross-section, which is 73,3 %.

IDEA Concrete does not calculate required reinforcement area to carry the external load actions, but it calculates the capacity of the whole reinforced cross-section. Results are compared in the following table:

Value	Reinforcement exploitation [%]	Cross-section exploitation [%]
Manual calculation	73,3	
IDEA Concrete		79,1

Results calculated in IDEA Concrete in the y-direction (90^{o} to the x-axis)

Capacity N-M-M

Angle between x-axis and checked direction : 90,0°

N_{Ed} [kN]	$M_{Ed,y}$ [kNm]	$M_{Ed,z}$ [kNm]	Type	Value [%]	Limit [%]	Check
-1785,70	1149,89	0,00	Nu-Mu-Mu	79,13	100,00	OK

1.1.7. Conclusion

Both methods are similar to each other. Both recalculate external loads to the reinforcement directions at different surfaces and the resulting tension normal forces are carried by reinforcement.

Differences in the normal forces calculation are caused by different calculation of internal forces arm z. The internal forces arm is calculated only from reinforcement in y-axis direction in the manual calculation, IDEA Concrete includes into calculation both reinforcement directions. The internal forces arm value is 0,68 m in the manual calculation and 0,66 m in IDEA Concrete. If the value of concrete cover changes to 30 mm, in IDEA Concrete the value of internal forces arm changes to 0,68 m and the cross-section exploitation decreases to 74,4 %. Than the cross-section exploitation is almost identical.

All compared values are presented in following table. Difference between calculated values is given in percent, where100 % is value from manual calculation.

Value	Manual calculation	IDEA Concrete	Difference [%]
V_{Ed} [kN/m]	39,4	39,4	0
$V_{Rd,c}$ [kN/m]	471,0	544,4	15,5
$n_{Edx,upper}$ [kN/m]	-1104,3	-1111,3	0,6
$n_{Edx,lower}$[kN/m]	-600,2	-593,4	1,1

$n_{Edy,upper}$ [kN/m]	-2607,4	-2653,8	1,7
$n_{Edy,lower}$[kN/m]	821,7	868,1	5,6
Exploitation	73,3	79,1	7,9

2. Literature

[1] Sekanina, D., *Interakce předpjatých konstrukcí v kontaktu s podložím*, PhD. thesis, Katedra konstrukcí, Fakulta stavební VŠB TU Ostrava, 2010

[2] ČSN EN 1992-2 (73 6208) Eurokód 2: Navrhování betonových konstrukcí - Část 2: Betonové mosty - Navrhování a konstrukční zásady