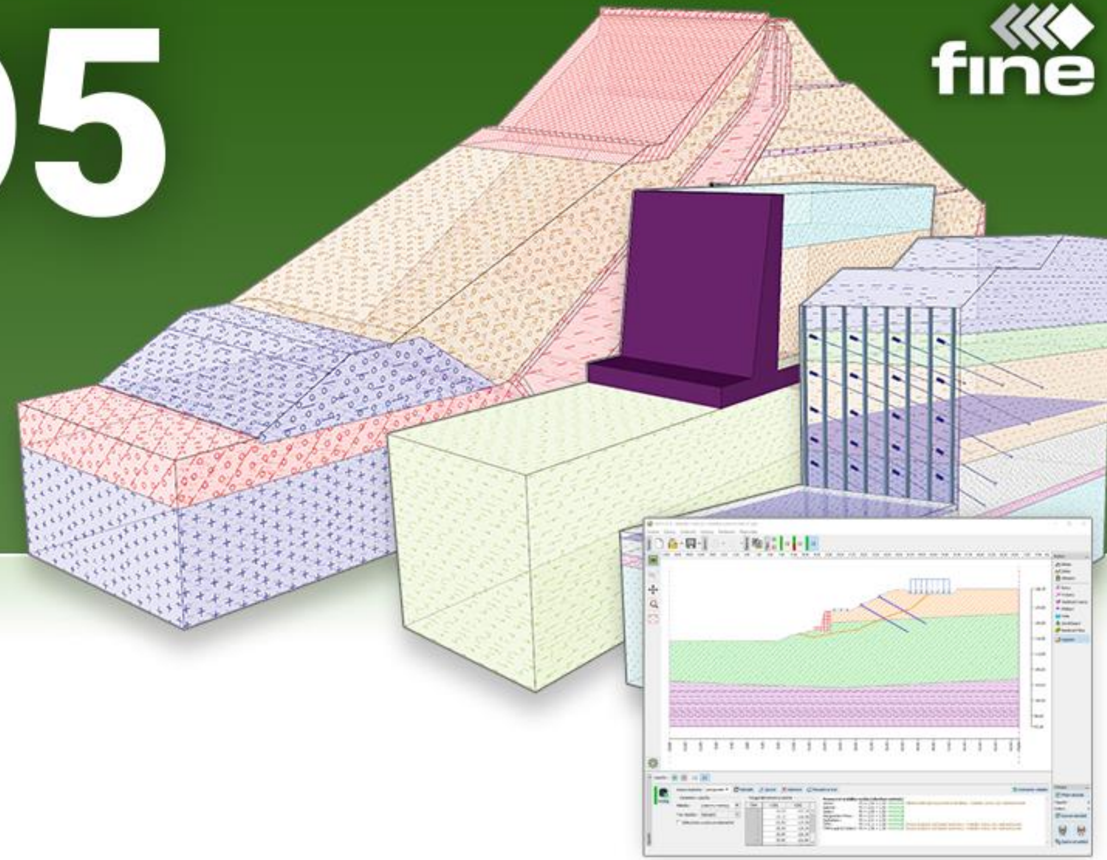
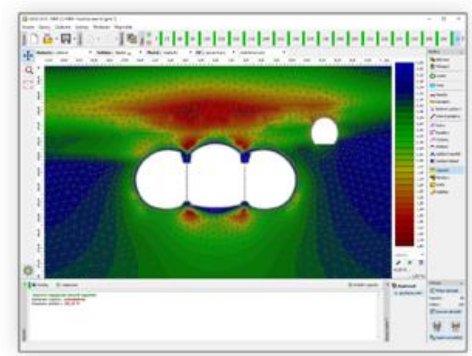
GEO5

Geotechnical Software

fine



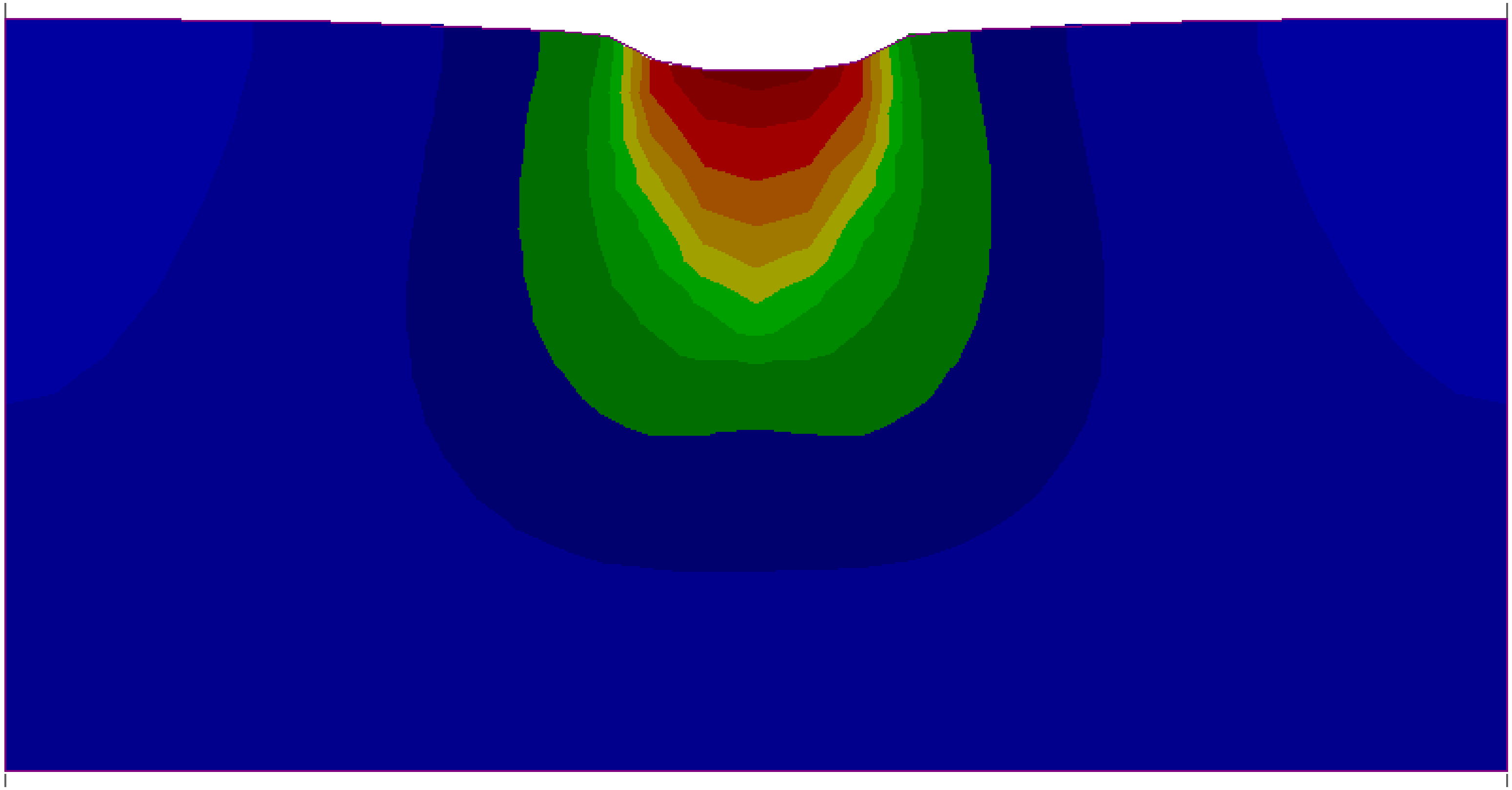


FEM – Fundamentals

Tomáš Janda

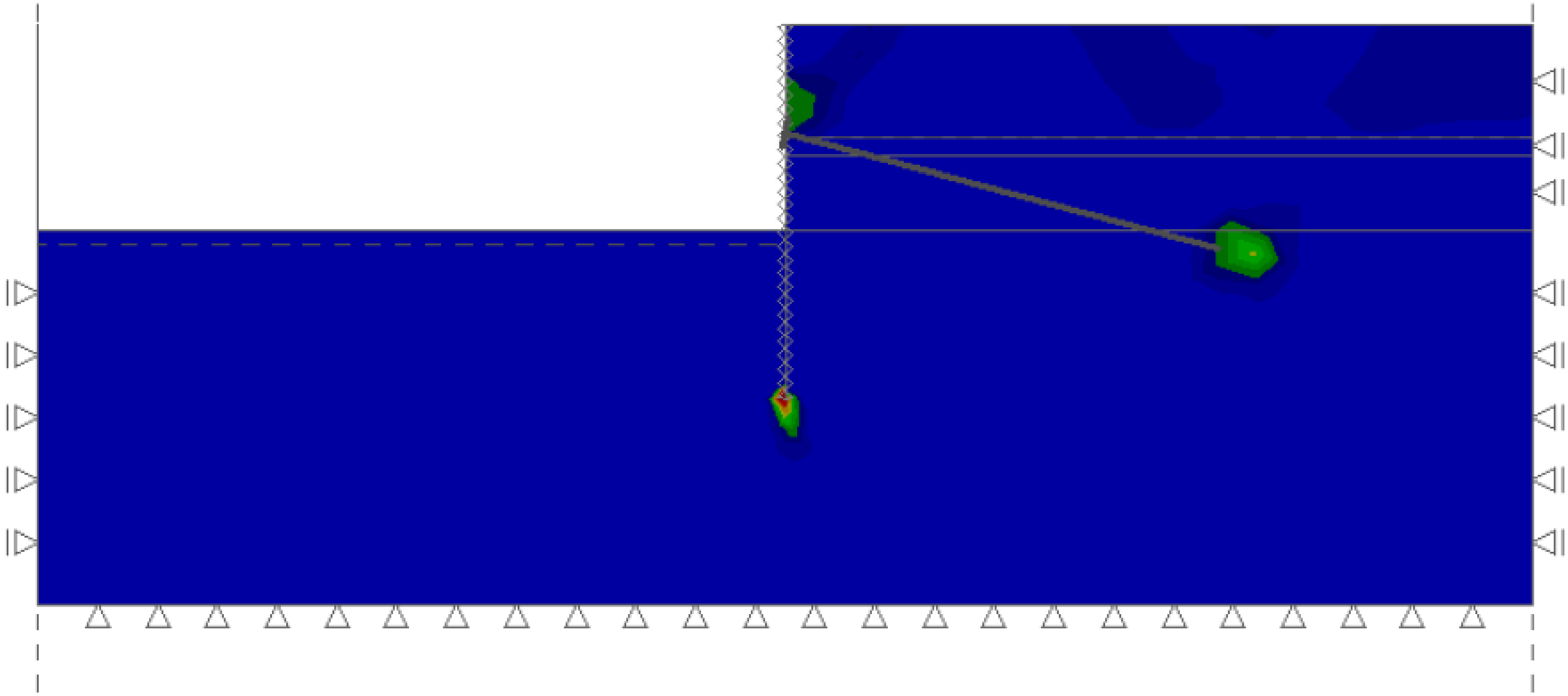
Motivation

- Settlement



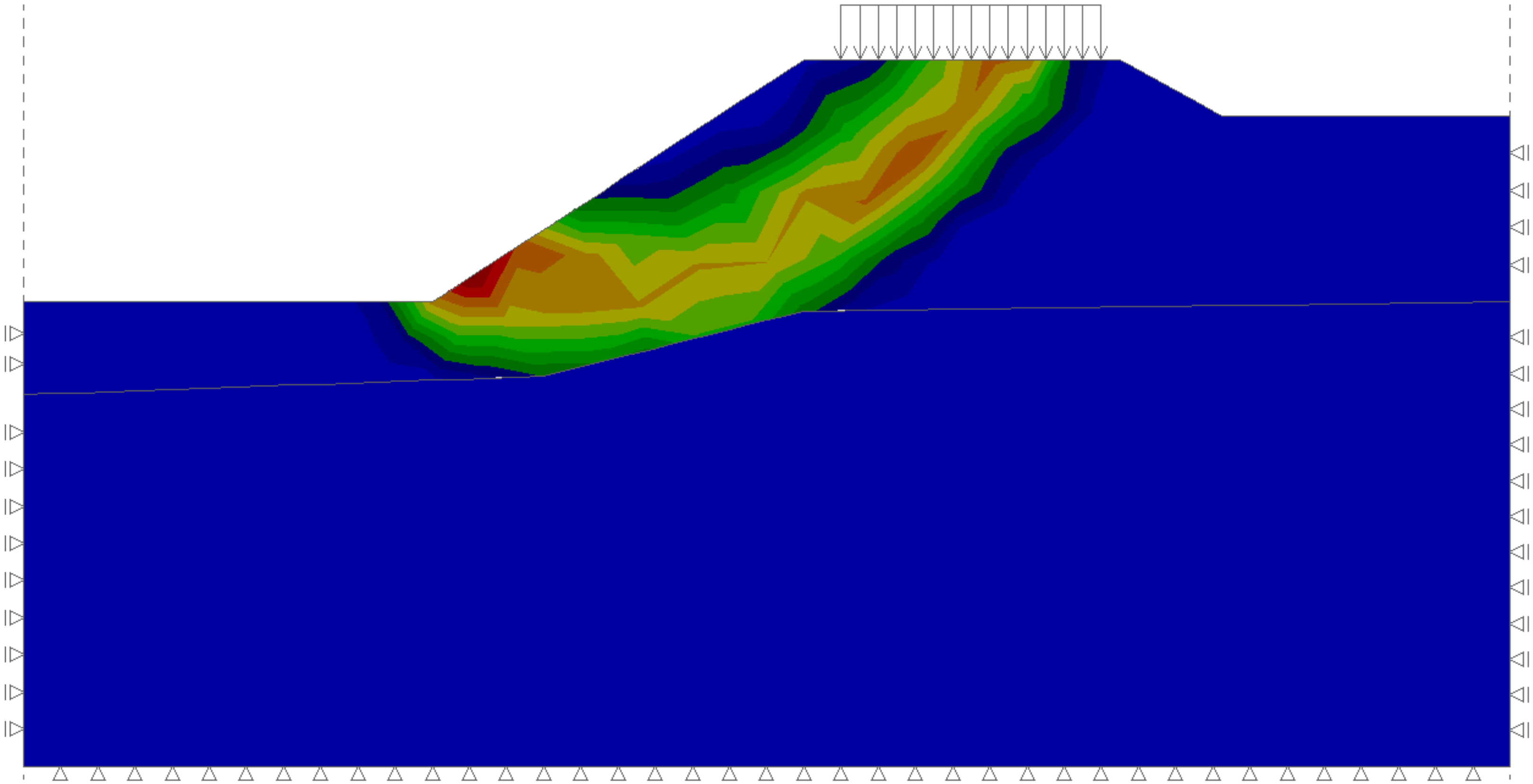
Motivation

- Retaining walls



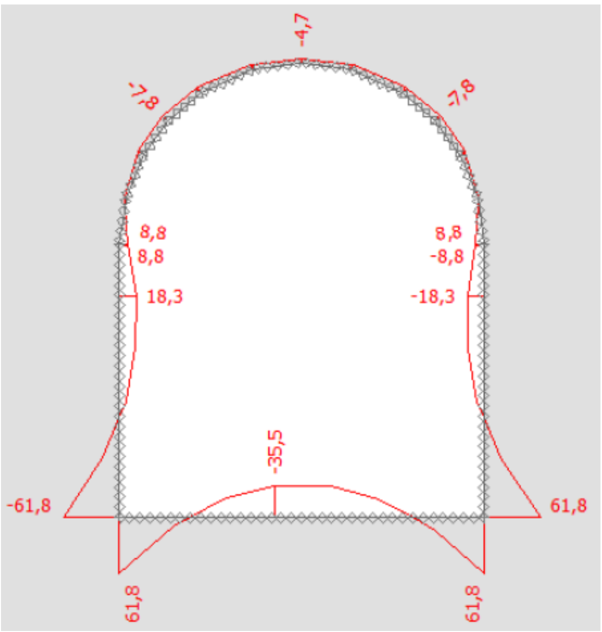
Motivation

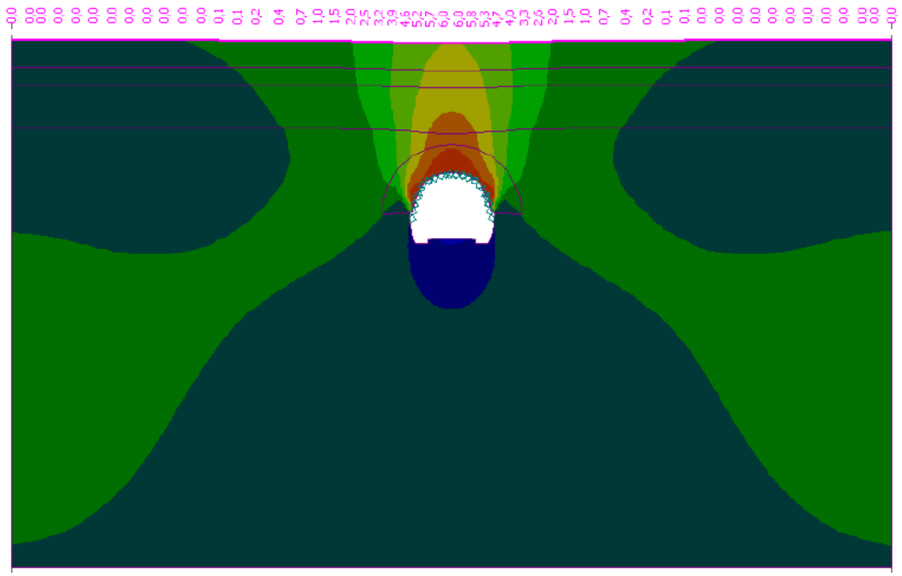
- Slope stability

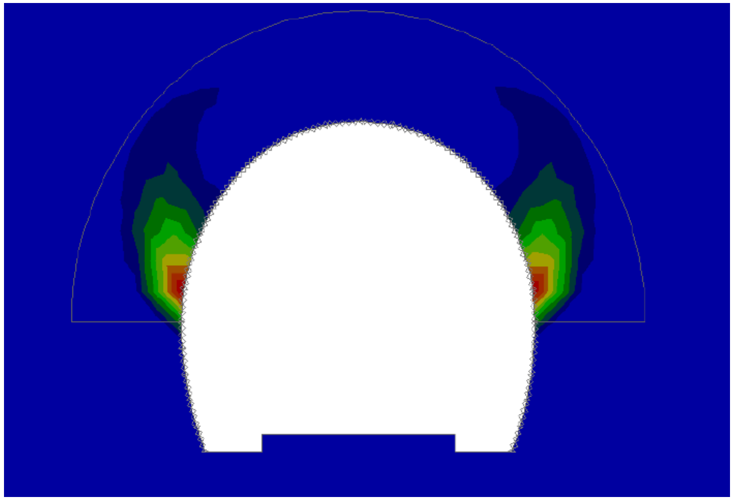


Motivation

- Tunnels

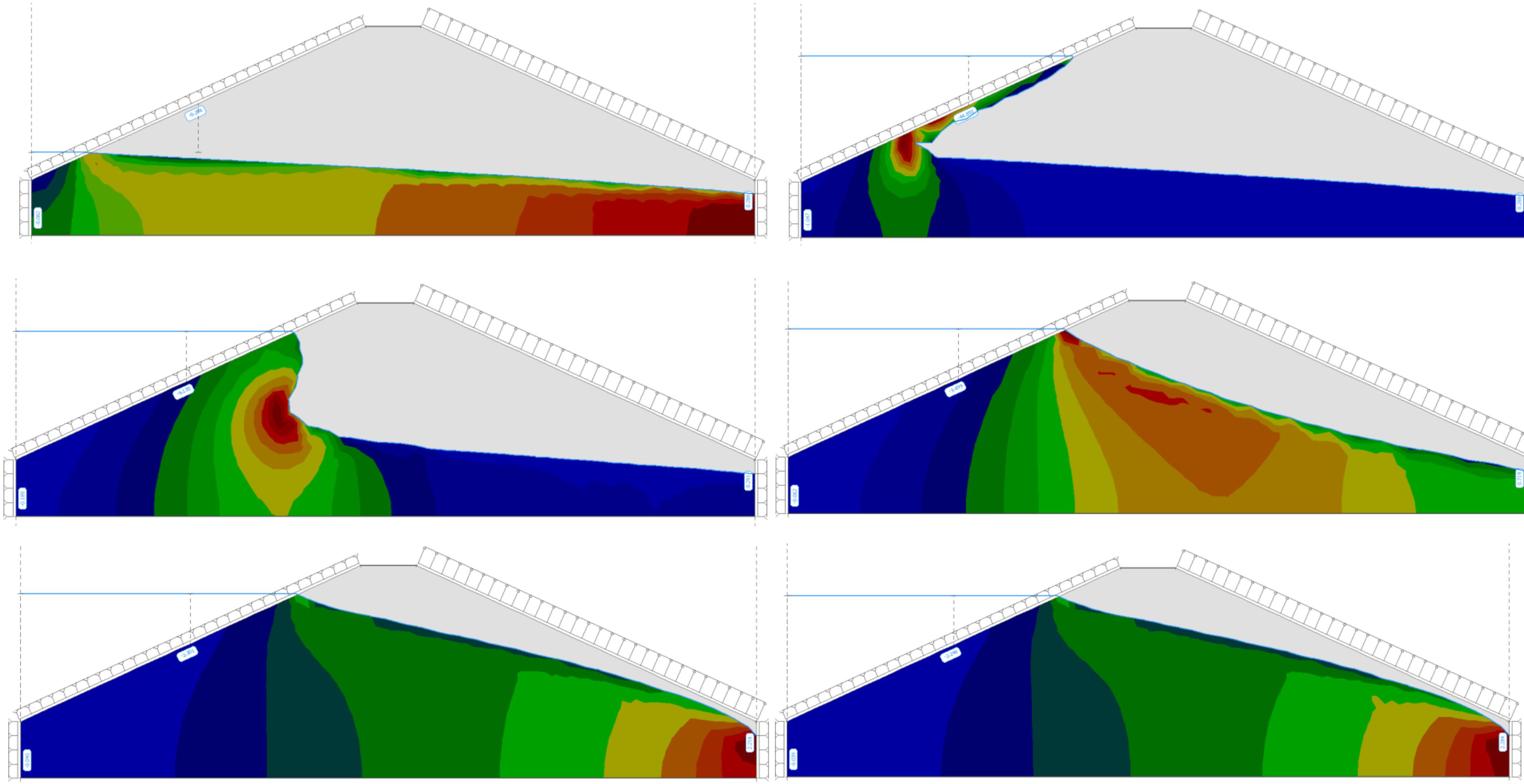






Motivation

- Transient water flow through an earth dam

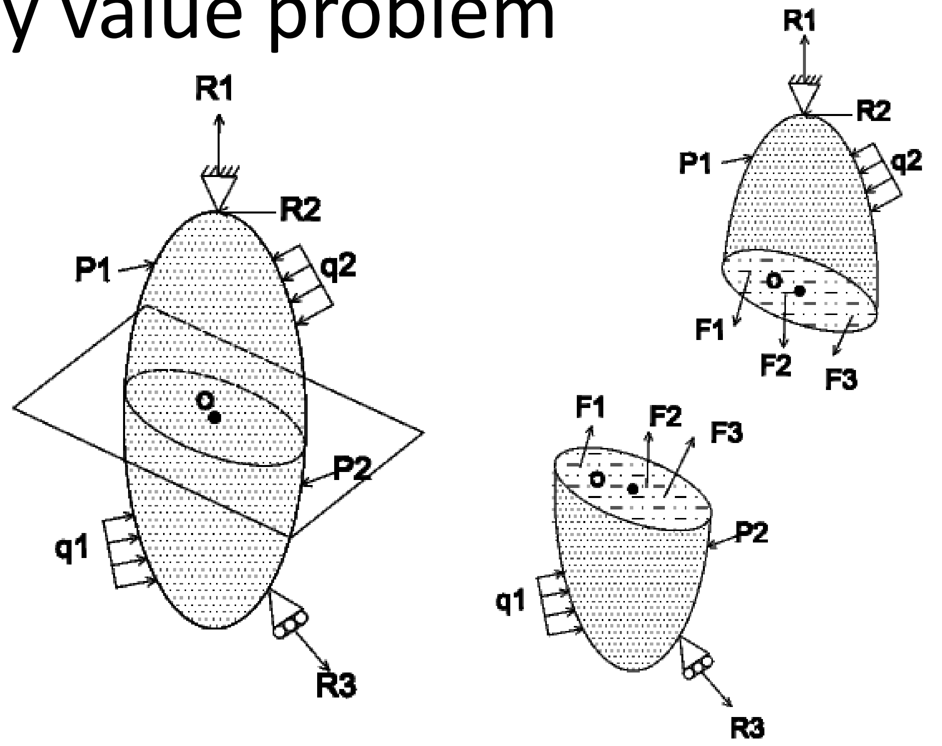


Outline

- 2 slides of theory
- Advantages vs. limitations
- **Essential terms** and principles
- Universal rule of FEM modeling
- FE mesh
- Stability

GEO 5 FEM - Theory

- Finite Element Method
  - Gives approximate solution to boundary value problem for partial differential equations.
- Boundary value problem



GEO 5 FEM - Theory

- Partial differential equation (PDE)
- Fulfilled at every point
- Various problems → various types of PDE
  - Mechanical
    - Static equations: $\boldsymbol{\partial\sigma} + \boldsymbol{X} = \boldsymbol{0}$
    - Stress strain relationship: $\boldsymbol{\sigma} = \boldsymbol{D}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_0)$
    - Geometric equations: $\boldsymbol{\varepsilon} = \boldsymbol{\partial}^T\boldsymbol{u}$
  - Water flow
    - Continuity equations: $n\frac{\partial S}{\partial t} + \nabla[nS\boldsymbol{v}^w] = m$
    - Darcy law: $nS\boldsymbol{v}^w = -\frac{K_r}{\gamma_w}(\nabla p - \gamma_w \boldsymbol{i}_g)$

FEM vs. Design codes

FEM analysis approximate the *physics* behind the behavior of the structure

Advantages

- Arbitrary geometry
- Multiple phenomena
- Very flexible models
- Helps you to understand the behavior
- Universal principles

Limitations

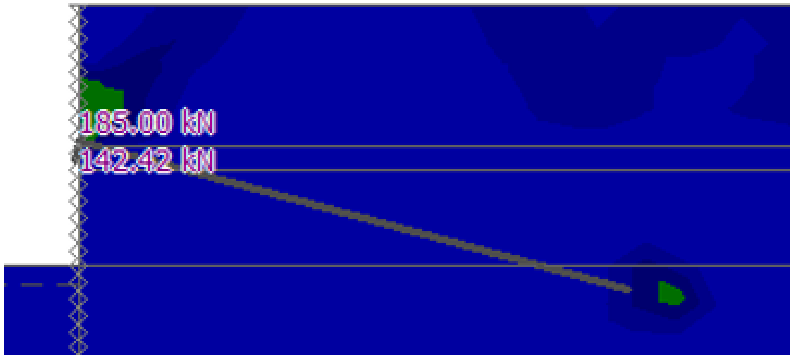
- No safety coefficients
- No design/characteristic values
- No design standards
- Not always on safe side
- Simplifications have to be made

Essential treminology

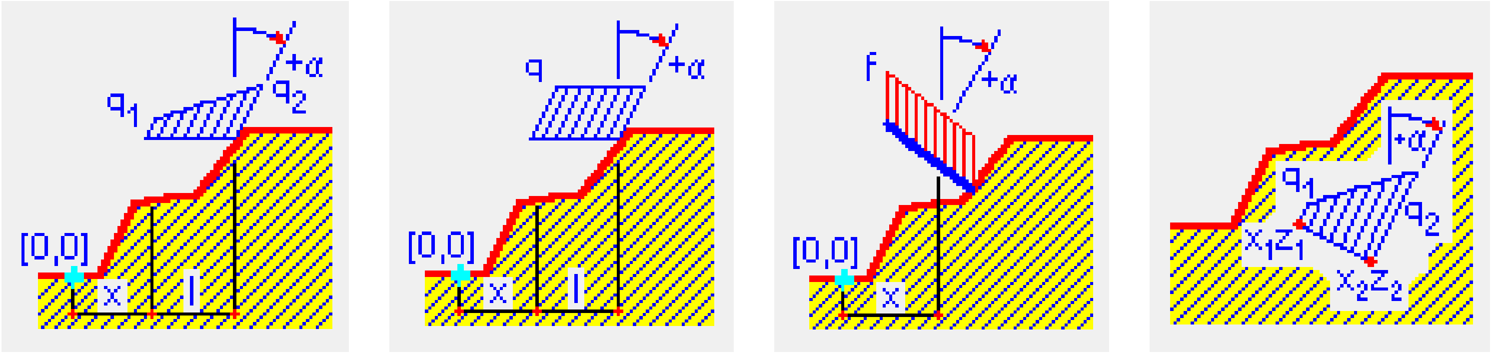
- Force, displacement, stress, strain, pressure
- Stiffness, strength
- 2D, plane strain, axisymmetry
- Boundary conditions
- Construction/calculation stages
- First stage, initial geostatic stress
- Groundwater

Force, loading, surcharge

- Units [kN]

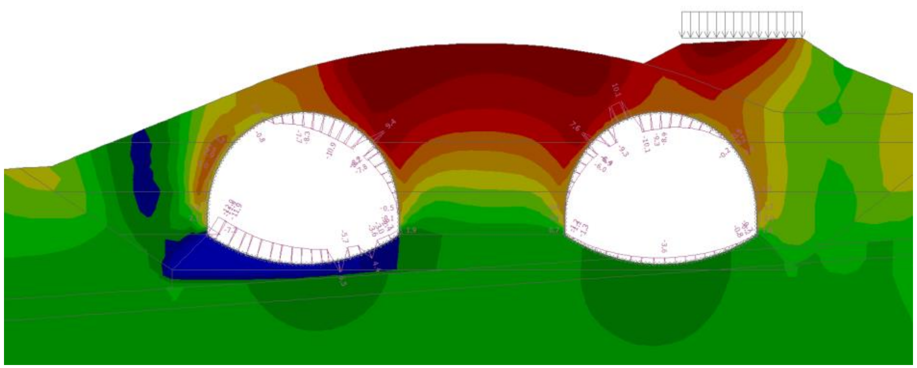


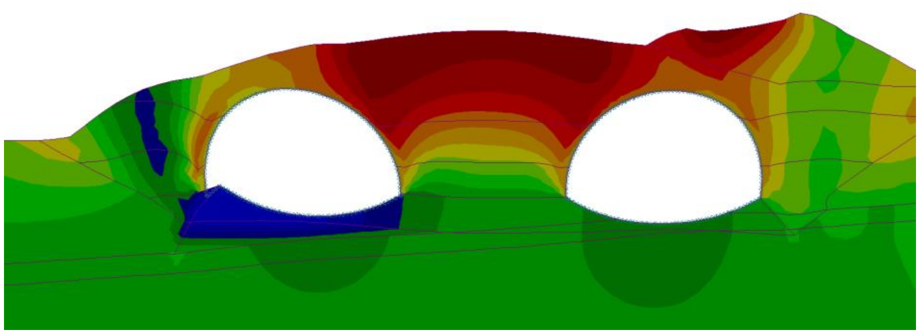
- Force per running, square or cubic meter
- Distributed loading, specific weight, …
- Units: [kN/m, kN/m², kN/m³]



Displacement

- Change in position
- Vector quantity: d_x, d_z
- Units [mm]
- Vertical displacement = settlement

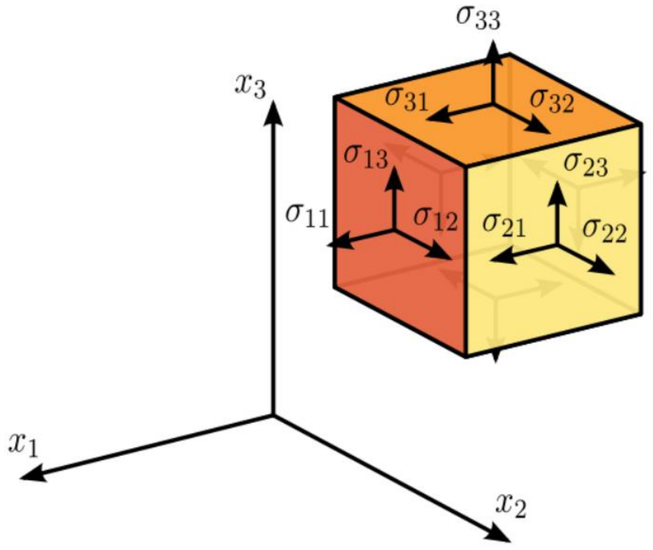




- I will avoid using the term “deformation”

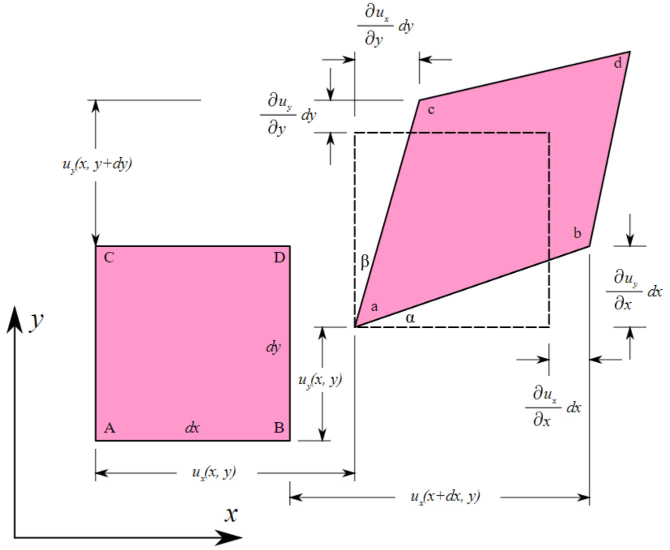
Stress σ

- Quantity at material point level
- Units [kPa]
- Tensorial – it has components
  - Normal stress σ_x, …
  - Shear stress τ_{xz}, …
- Invariants
  - Mean stress σ_m
  - Equivalent deviatoric stress J



Strain ε

- Change in shape and size of small piece of material
- Units [%]
- Tensor quantity
  – Normal strain ε_x, …
  – Shear strain γ_{xz}, …
- Invariants
  – Volumetric strain ε_v
  – Equivalent deviatoric strain E_d



Stiffness

- At material level
  - Hooke's law of elasticity

$$\boldsymbol{\sigma} = \boldsymbol{D}\boldsymbol{\varepsilon}$$

  - Stiffness parameters
    - Young's modulus E [kPa]
    - Shear modulus G [kPa]
    - Poisson's ratio ν [–]

- Structure with low stiffness will deform excessively
- Stiffness is different from strength

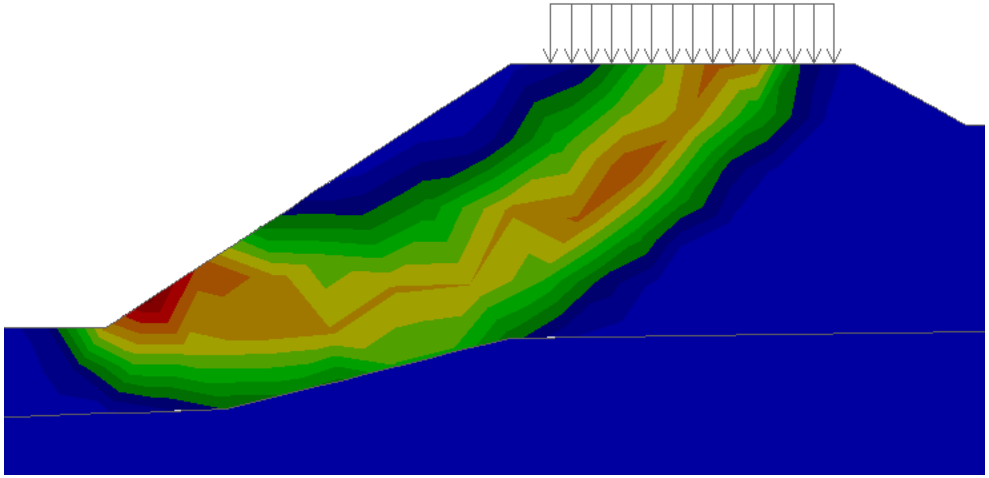
Strength

- Stress level at failure
- Strength depends on material model
  - Elastic model – infinite strength
  - Mohr-Coulomb:
    - Angle of internal friction φ
    - Cohession c
- Low strength → high plastic strain, unstable structure, failure

Plasticity

- **Elastic strain** – reversible
  - Deformation of the soil's grains itself
- **Plastic strain** – irreversible
  - Change in arrangement of the soil's grains
- High plastic strains

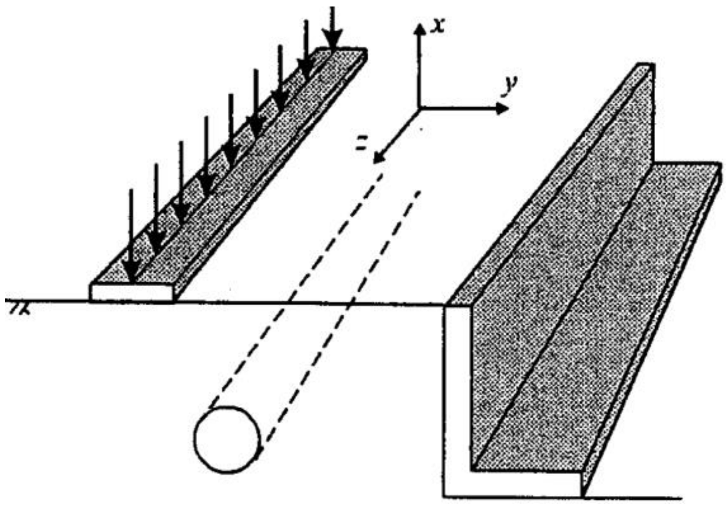
  → problematic areas
- Equivalent deviatoric plastic strain $E_{d,pl}$



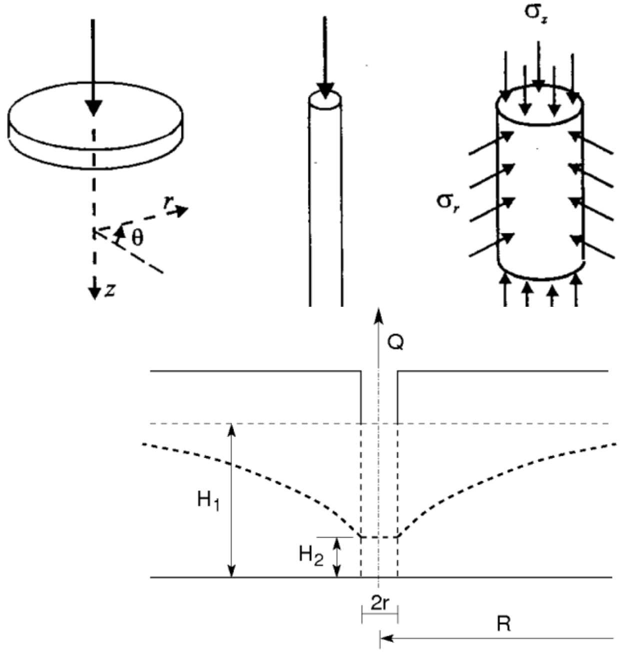
2D analysis

- Real world is 3D
- Most real word structures can be represented with 2D model (pile walls, row of anchors, …)

Plane-strain

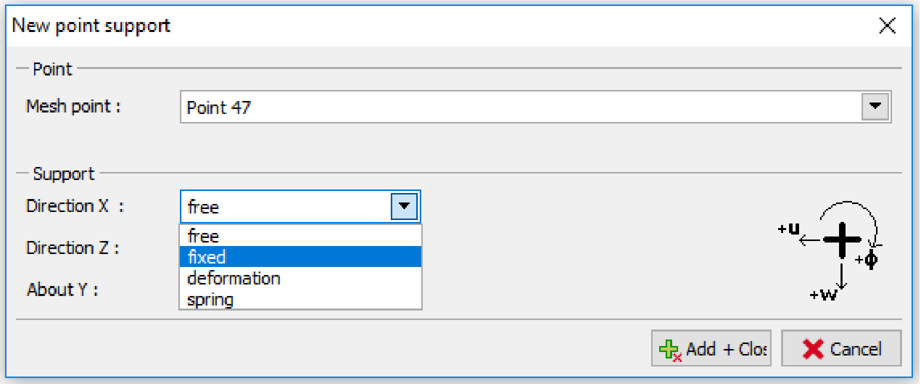


Axisymmetry

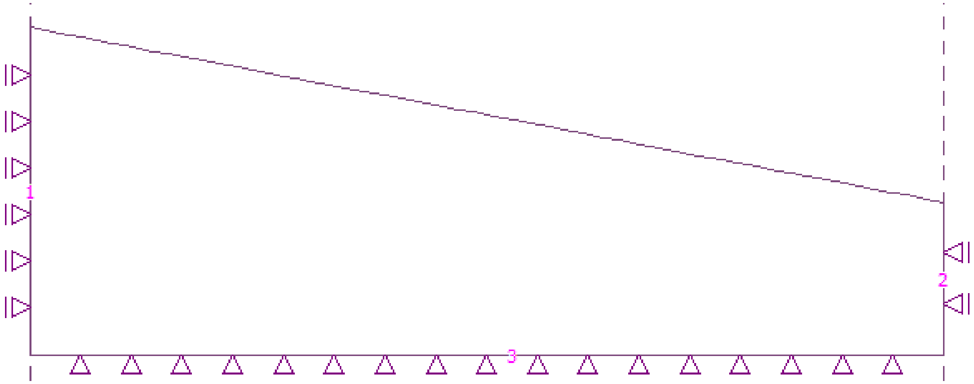


Boundary conditions

- Prevent or prescribe displacement or rotation
- „Supports“
- **Point supports**

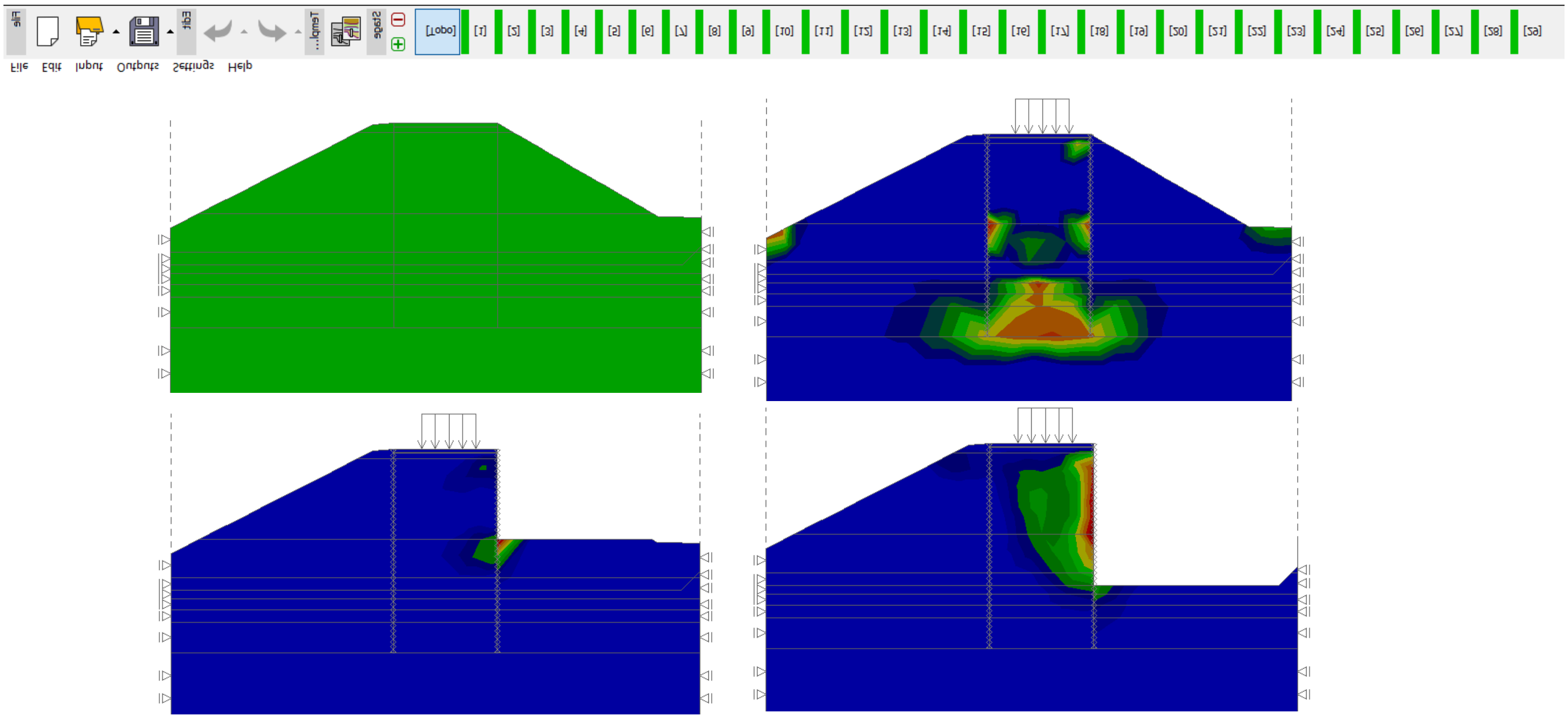


- **Line supports**
  - Added automatically at the bottom and lateral boundaries
  - Might be edited by user



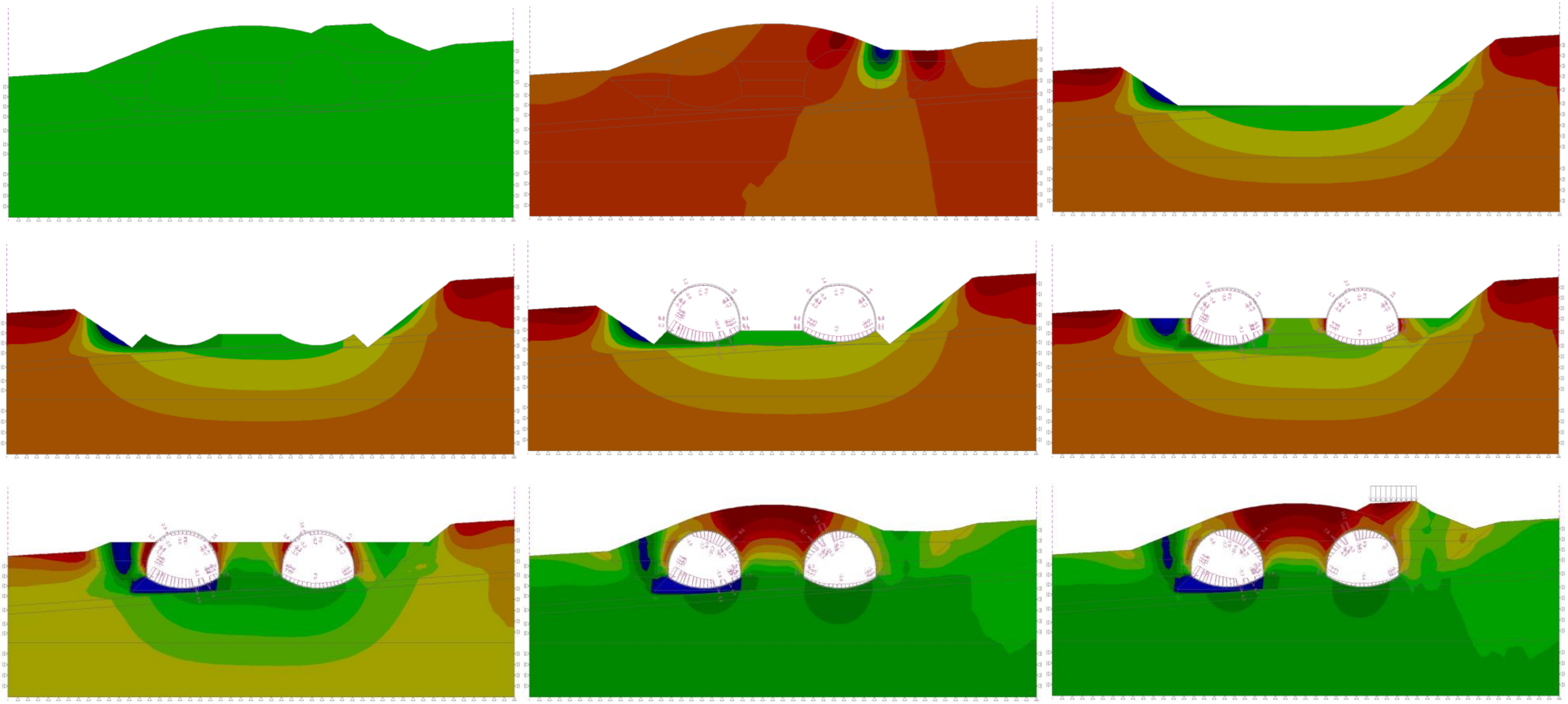
Construction/calculation stages

- Incremental solution strategy
- Stages are not variants (order matters)



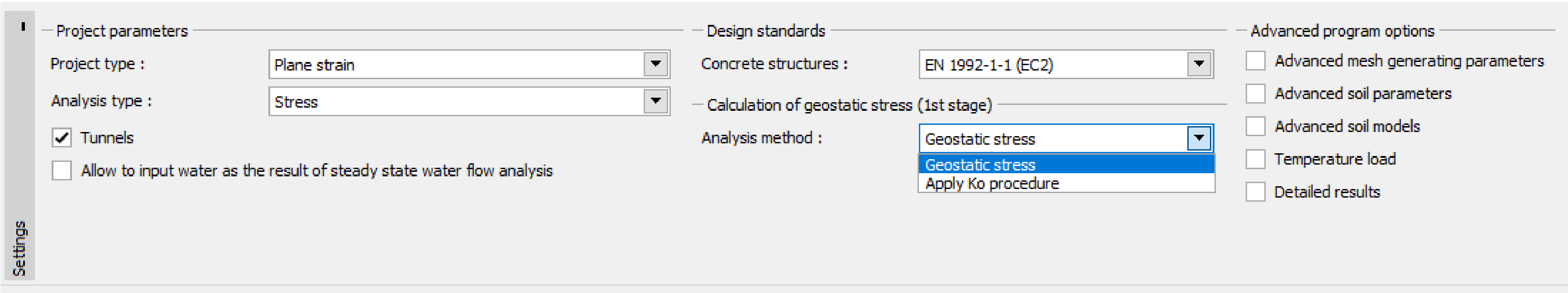
Construction stages

Cut and cover tunnel



First stage

- Special stage to create initial geostatic stress
- To ways to do that:
  - Load the domain with gravity
  - Get vertical stress from unit weight and horizontal stress from coefficient of lateral pressure K_0



- Displacement is zeroed in firs stage (undeformed state)

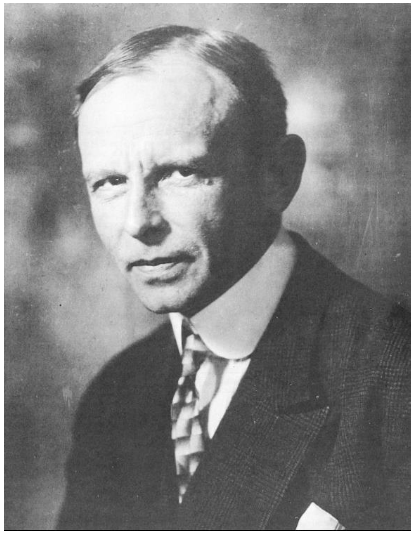
Groundwater

- **Karl von Terzaghi** (1883–1963)

  Austrian civil and geotechnical engineer

  known as the "father of soil mechanics”

  born in Prague



- Terzaghi theory of effective stresses

$$\sigma = \sigma' + u$$

- All observable changes in soil are result on change in *effective* stress

Groundwater

- No water
- Prescribed GWT
- Detailed pore pressure field

- Materials
  - Drained – permeable soil, pore pressure excess will dissipate
  - Undrained pore pressure excess is calculated, no volumetric strain

Golden rule of FEM modeling

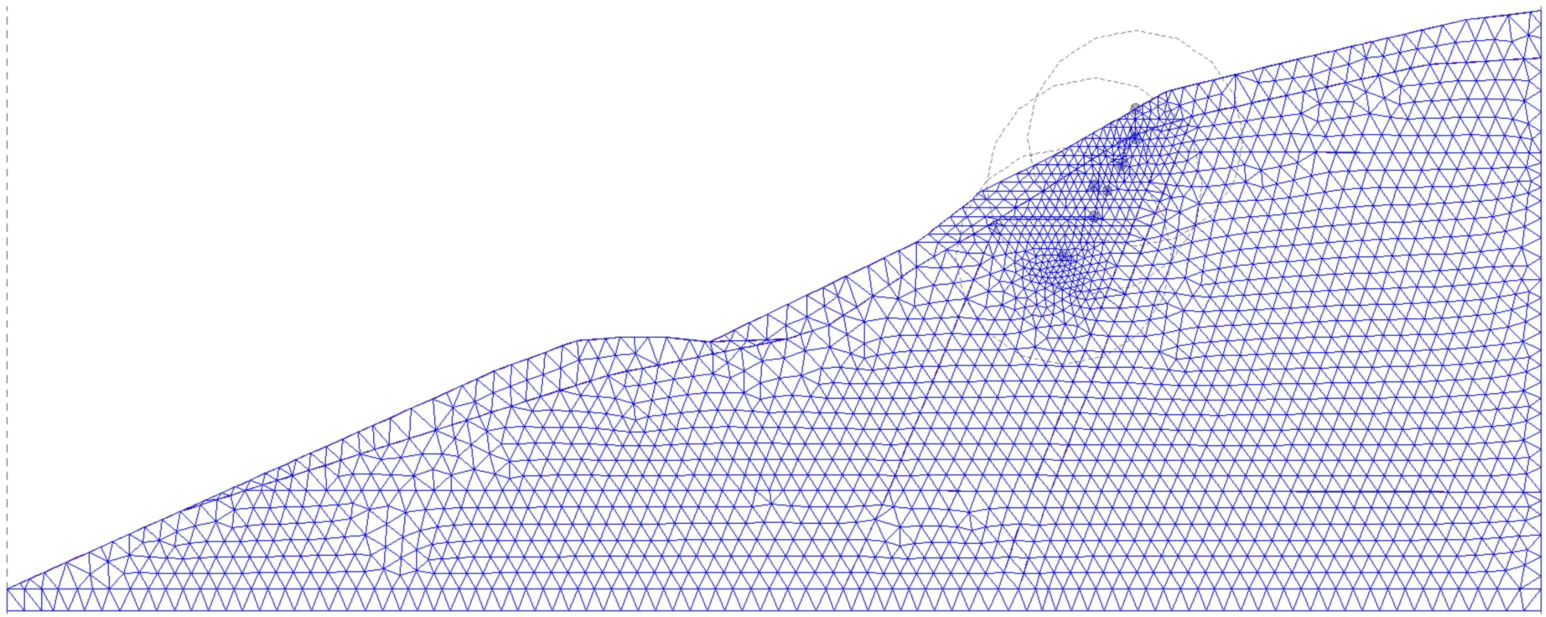
Make your model as simple as possible,
but not simpler

(3D → 2D, simple topology, material model)

Everything should be made as simple as possible,
but not simpler–Albert Einstein

FE Mesh

- Avoid pointy triangles
  - Refine mesh adequately
  - Modify topology



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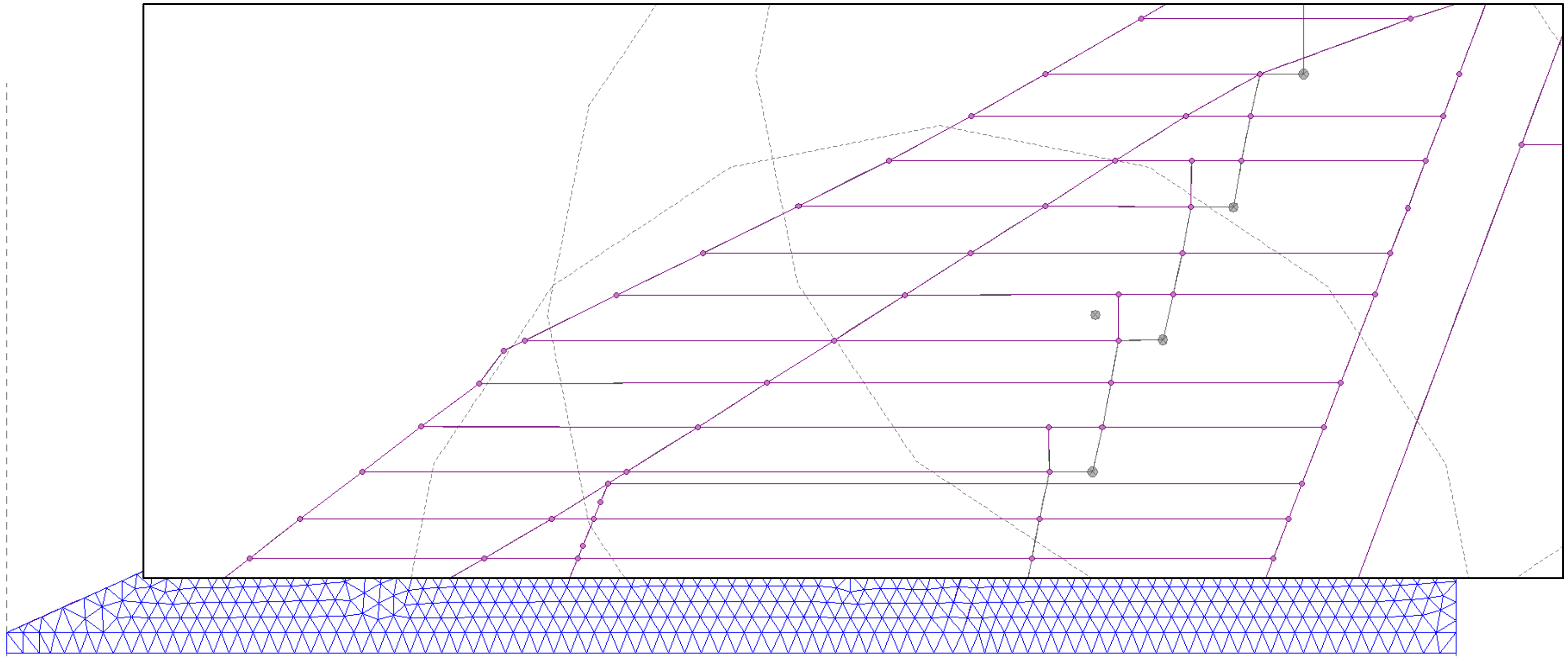

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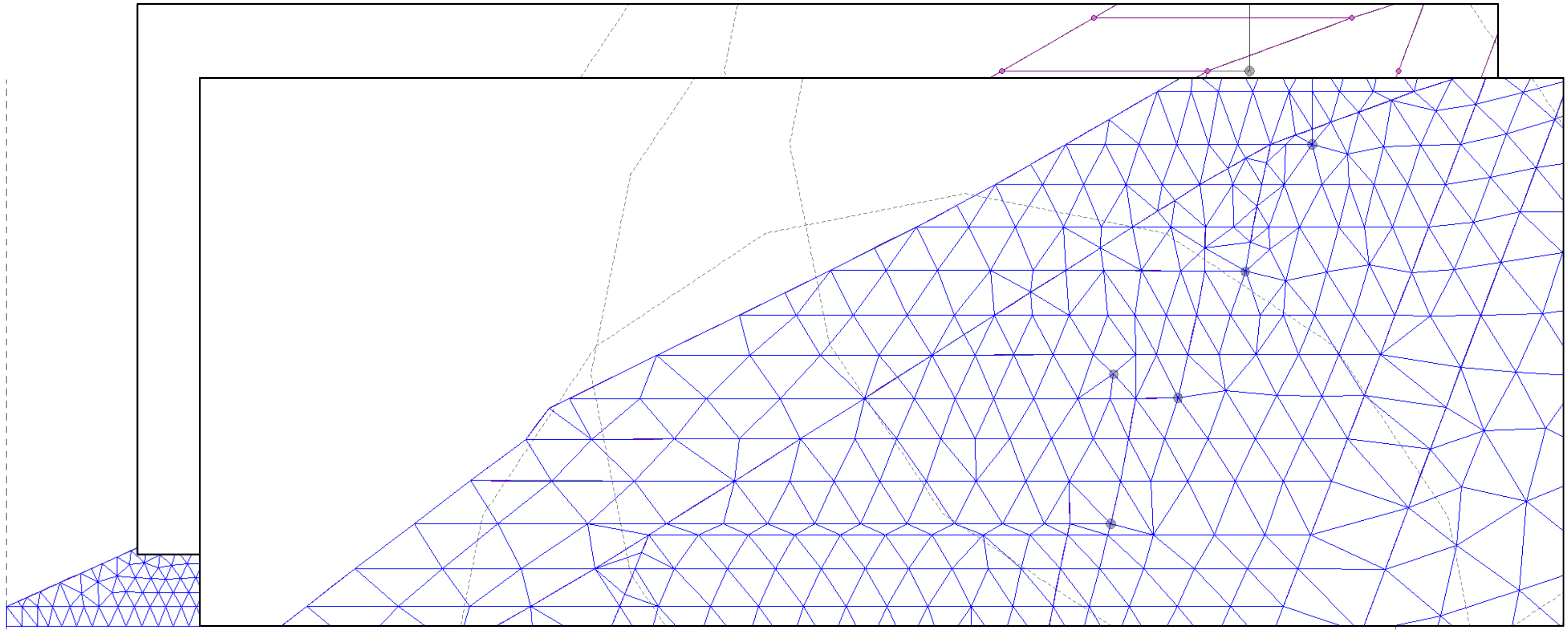
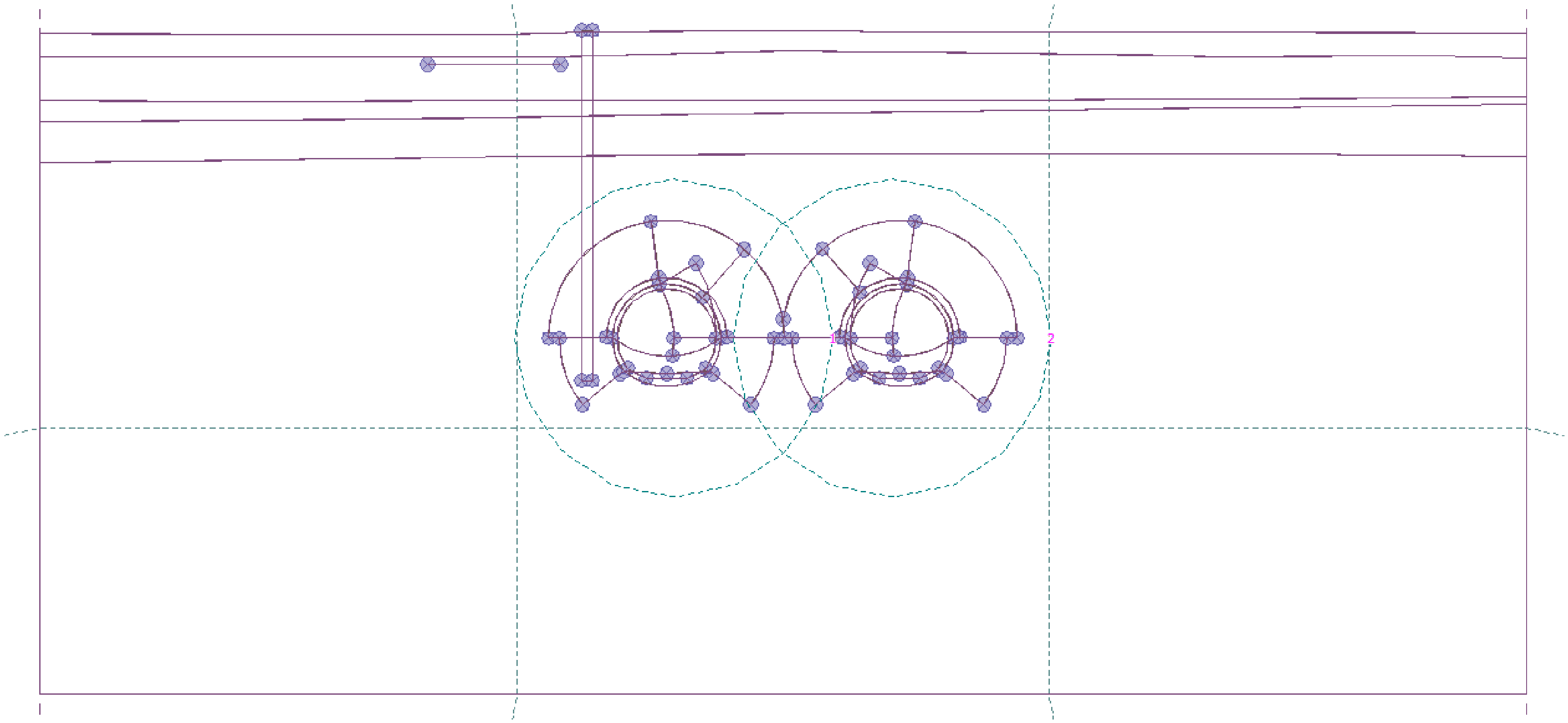


FE Mesh

- Overly complicated topology



FE Mesh

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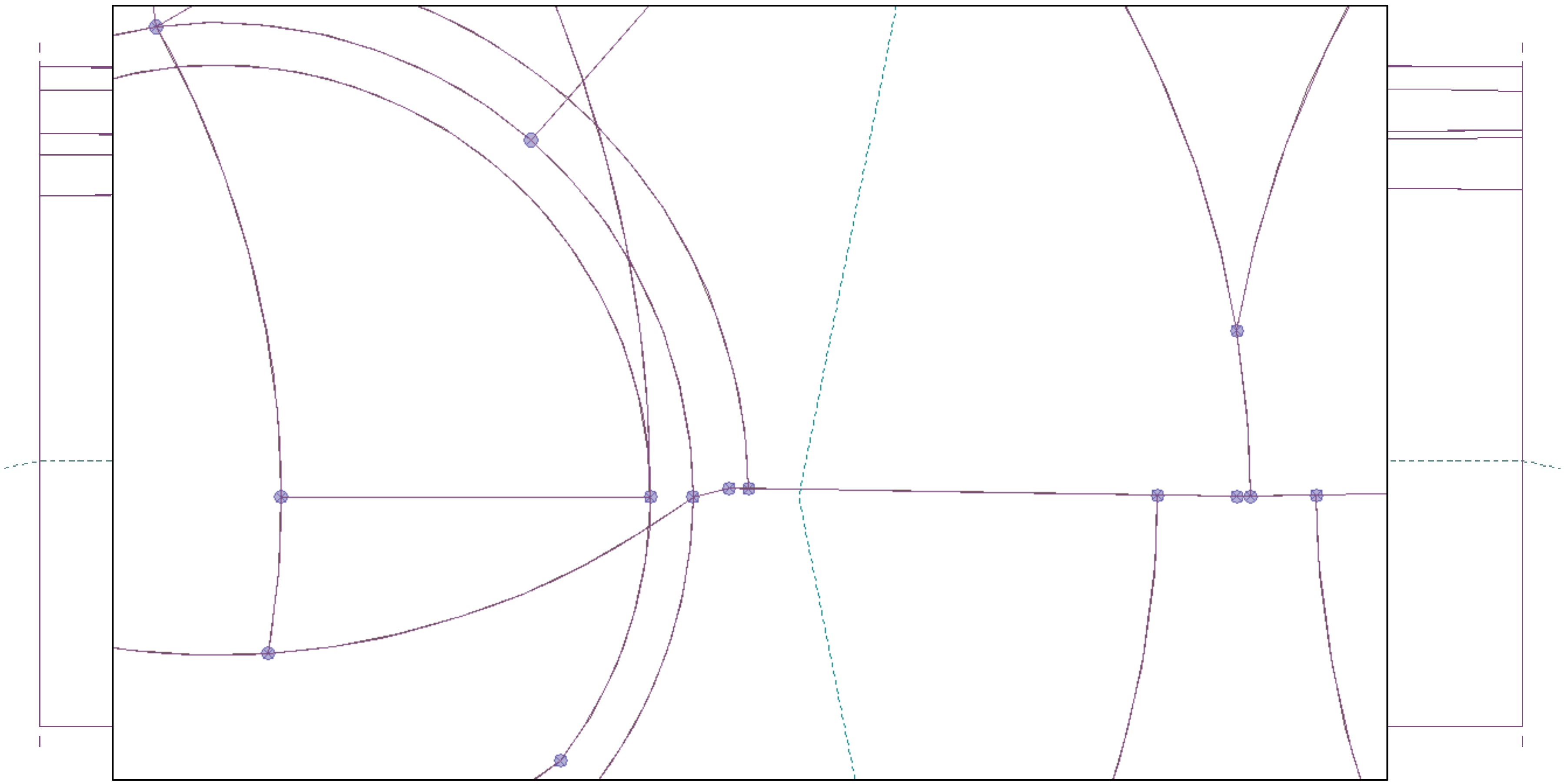
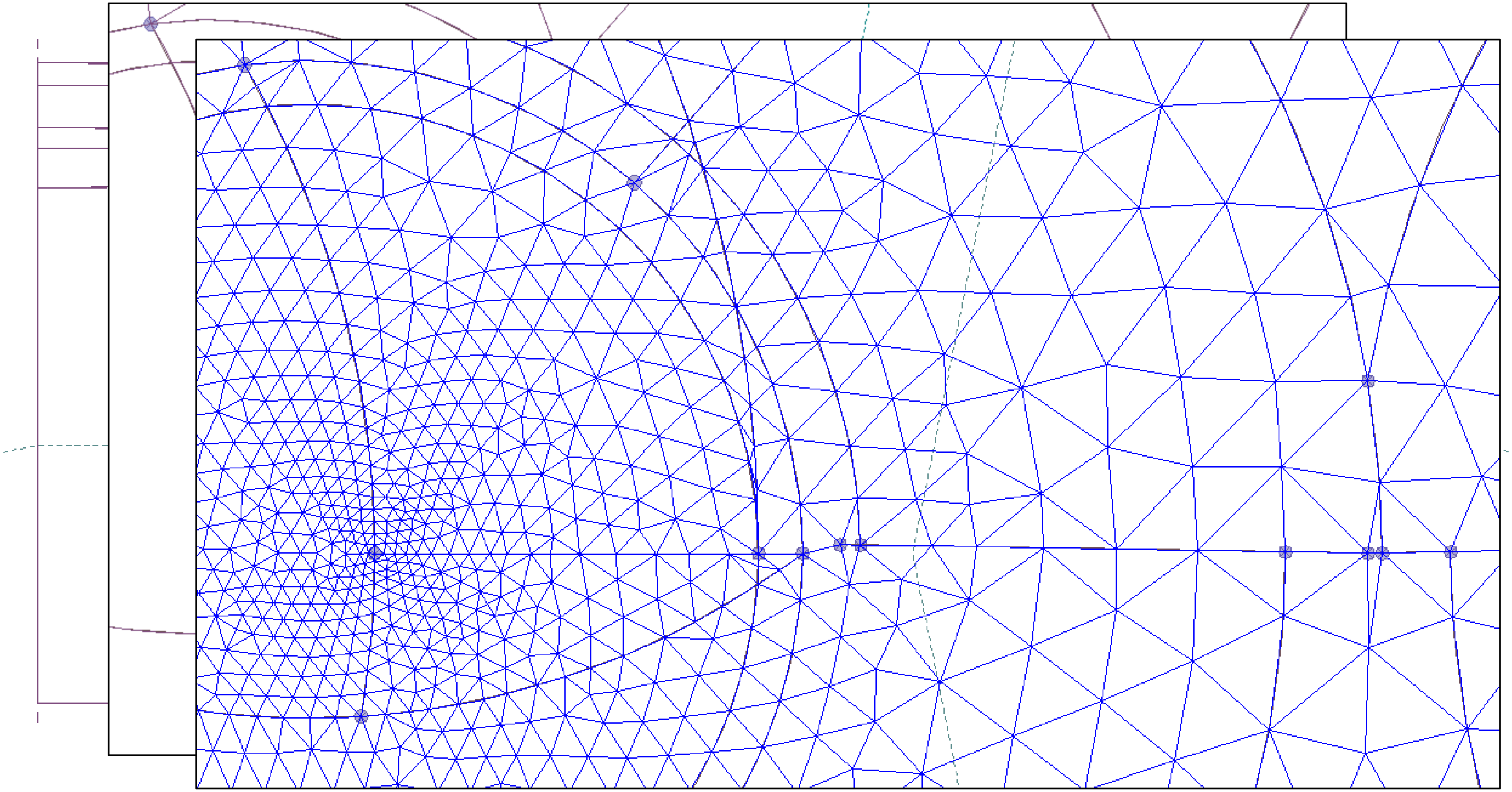


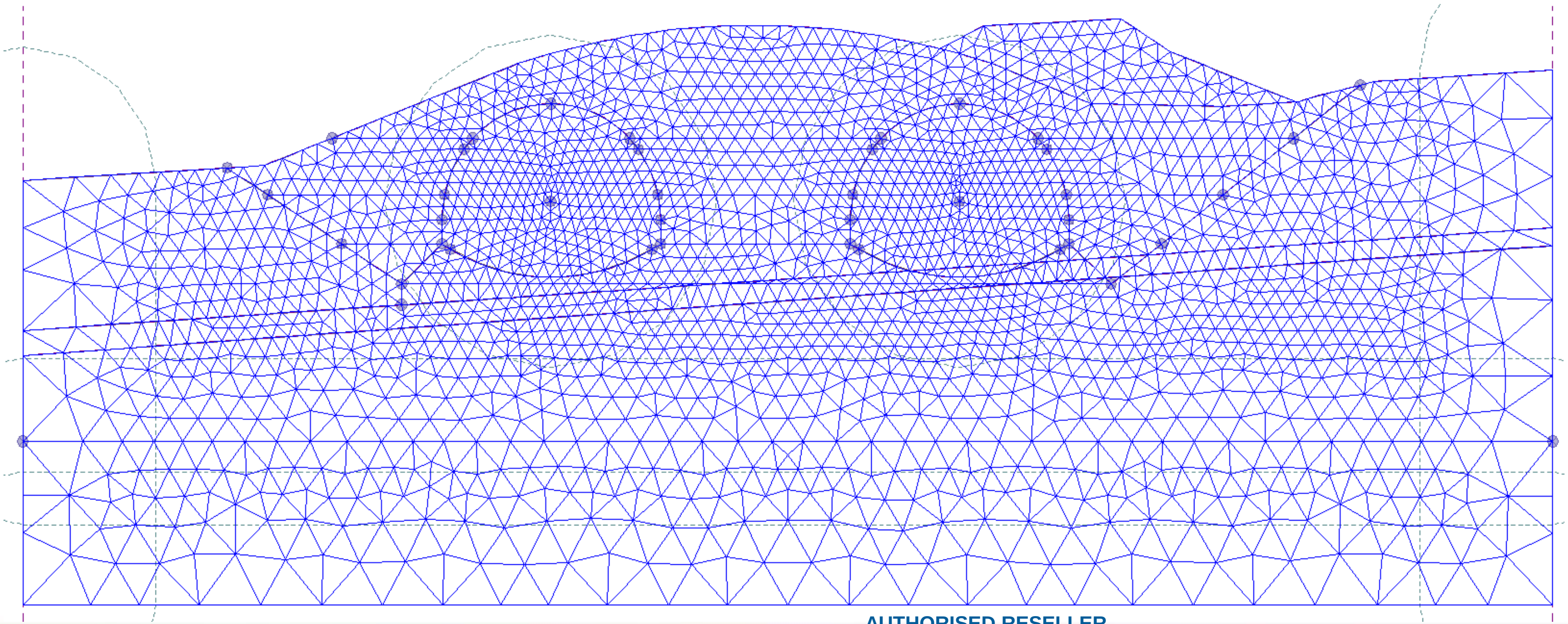
FE Mesh

- Overly complicated topology



FE mesh

- This mesh looks OK



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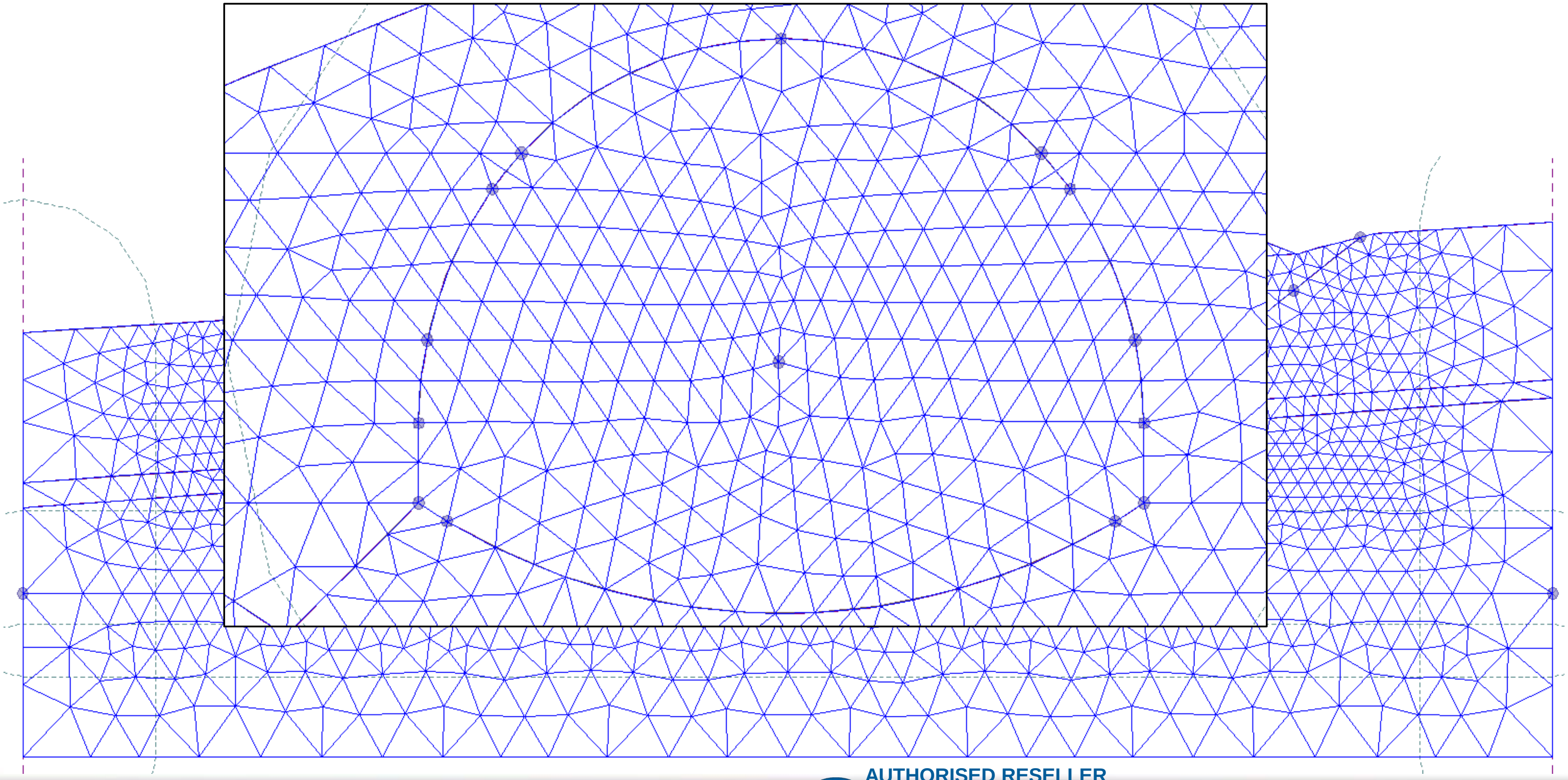


Stability analysis

- **Strength reduction method**

  Analysis converge → equilibrium → stable structure

  No convergence → unstable structure

- Strength parameters φ and c are decreased to get unstable state
- Factor of safety

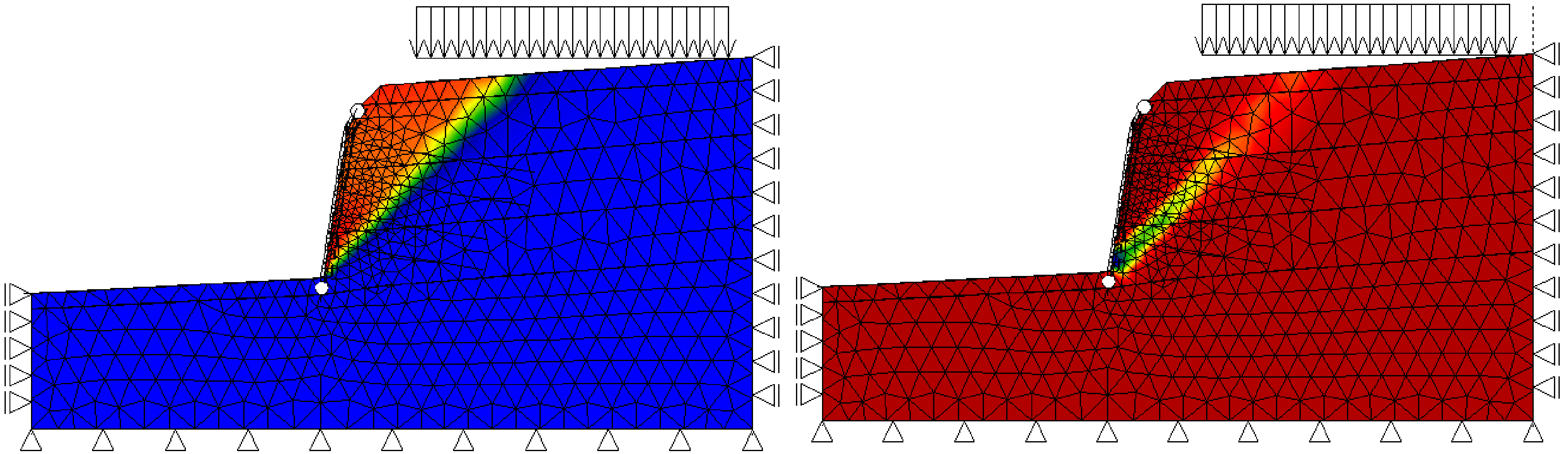
$$FS = \frac{\varphi_s}{\varphi_p}$$

φ_s … real angle of internal friction

φ_p … reduced angle of internal friction at failure

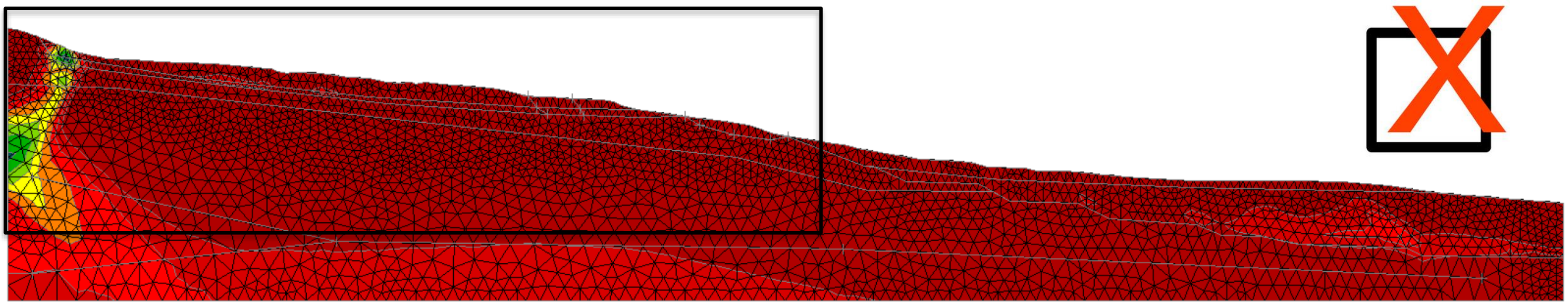
Stability analysis

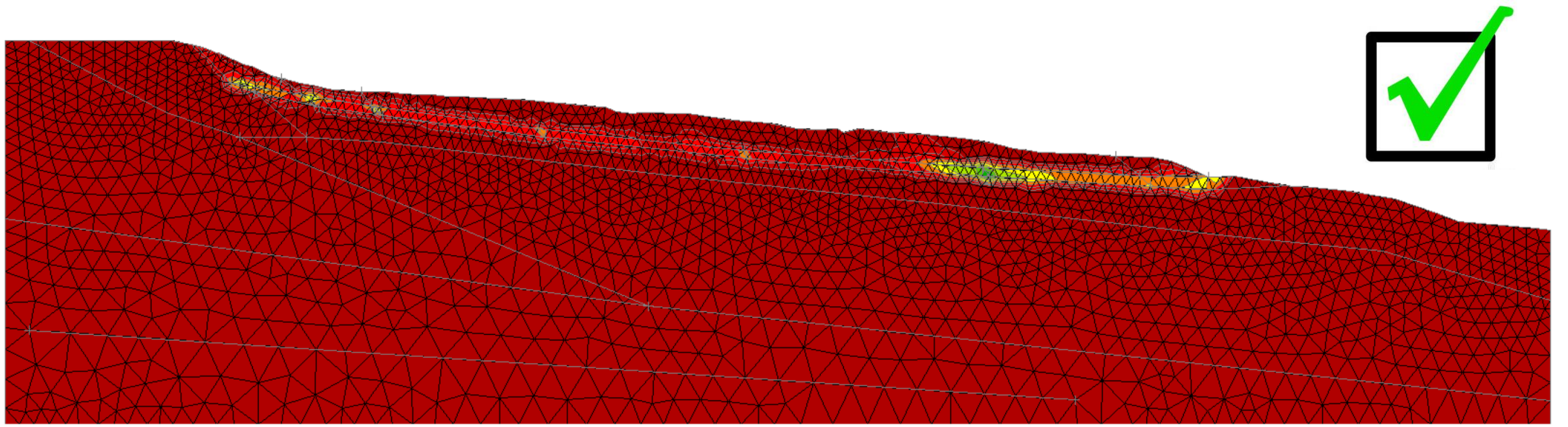
- Results
  - Examine the equivalent deviatoric plastic strain $E_{d,pl}$
  - Shear band = slip surface
  - Not clear failure mechanism → not reliable FS



Stability

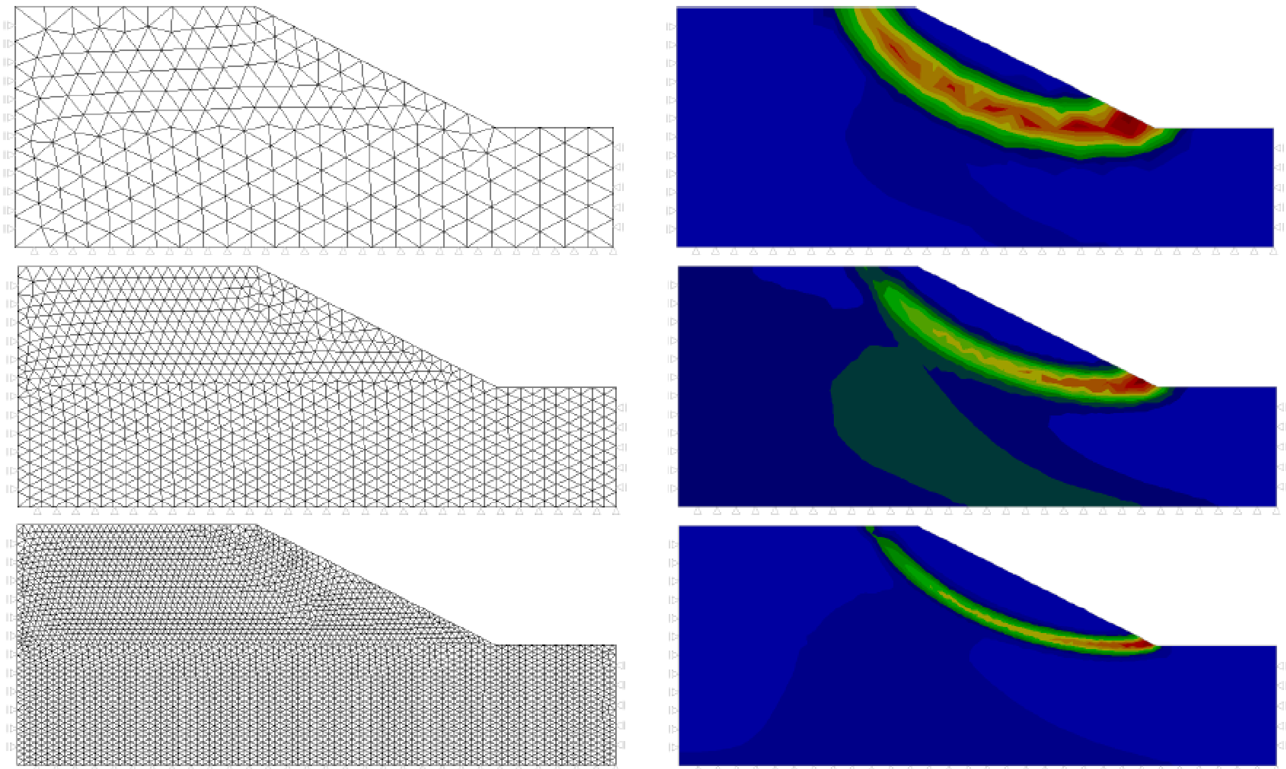
- Not clear failure mechanism → not reliable FS
- Distribution of $E_{d,pl}$





Stability

- Finer mesh
  - → slightly smaller FS
  - → slightly narrower plastic zone



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EISEKO COMPUTERS S.R.L.

Viale del Lavoro, 17
37036 S. Martino B.A. (VR)
tel. + 39 045 8031894
www.eiseko.it

Tel. 045 8031894

support@eiseko.com

Assistenza Remota