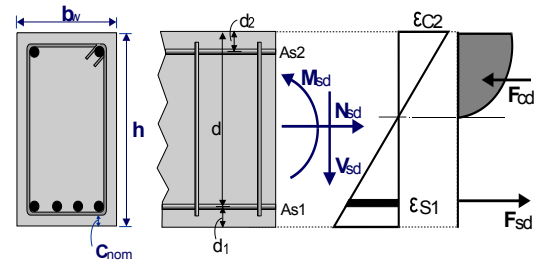


Example Beams

1. BEAM-001

Design of beam section for bending, shear and axial force
(EC2 EN1992-1-1:2004, EC0 EN1990-1-1:2002)

bx**h**=0.250x0.500 m, **M**sd=150.00 kNm,
Vsd= 40.00 kN, **N**sd= 40.00 kN
Concrete-Steel class: C25/30-S500 (EC2 §3)
Concrete cover : Cnom=20 mm (EC2 §4.4.1)
 $\gamma_c=1.50$, $\gamma_s=1.15$ (EC2 Table 2.1N)



1.1. Dimensions and loads

Beam width **b**w=0.250 m, beam height **h**=0.500 m
Bending moment **M**sd=150.00 kNm, Shear **V**sd=40.00 kN, Axial force **N**sd=40.00 kN (tension)
Effective depth of cross section **d**1=Cnom+ø_s+1.1ø=20+8+1.1x10=39mm, **d**2=39mm, **d**=500-39=461mm

1.2. Ultimate limit state, design for bending with axial force

(EC2 §6.1, §9.2.1)

Dimensioning for bending: Allgower, G.-Avak, R. Bemessungstabeln nach Eurocode 2 für Rechteck und Plattenbalkenquerschnitte, In: Beton - und Stahlbetonbau 87 (1992)

Reinforcement for bending with axial force (only tension reinforcement is needed)
Msd=150kNm, **N**sd= 40kN, **b**w=250mm, **d**=461mm, **K**d=1.94 **x**/**d**=0.26 $\epsilon_c/\epsilon_s=3.5/9.9$ **k**s=2.58, **A**s1= 8.84cm²
Minimum longitudinal tension reinf., **A**s>=0.26**b**d·**F**ctm/**f**yk, (**A**s,min= 1.56cm²) (EC2 §9.2.1.1.1)
Maximum tension or compression reinf., **A**s<=0.04**A**c, (**A**s,max=50.00cm²) (EC2 §9.2.1.1.3)

Longitudinal reinforcement: 6Ø14 (9.24cm²) (bottom)

1.3. Design against shear failure

(EC2 EN1992-1-1:2004, §6.2, §9.2.2)

Shear capacity without shear reinforcement **V**rdc (EC2 §6.2.2)
Vrdc=[**C**rdc·**k**·(100ρ_l·**f**ck)^{0.333}+**k**1·σcp]·**b**w·**d** (EC2 Eq.6.2.a)
Vrdc>=(**v**min+**k**1·σcp)·**b**w·**d** (EC2 Eq.6.2.b)
Crdc=0.18/γ_c=0.18/1.50=0.120, **f**ck=25.00MPa
k=1+(200/**d**)^{1/2} <=2, **k**=1.66, **k**1=0.15
ρ_l=**A**s1/(**b**w·**d**)=924/(250x461)=0.0080
σcp=**N**sd/**A**c=-1000x40.00/125000=-0.32N/mm²
vmin=0.035·**k**^{1.50}·**f**ck^{1/2} = 0.37N/mm² (EC2 Eq.6.3N)
Vrd,c(min)=0.001x(0.37-0.15x0.32)x250x461=37.11kN
Vrdc=0.001x[0.120x1.66x(0.80x25.00)^{0.333}-0.15x0.32]x250x461=56.79kN
Vsd=40.00 kN <= **V**rdc=56.79 kN, **V**sd<=**V**rdc shear reinforcement is not needed

Concrete strut capacity **V**rdmax (EC2 §6.2.3 Eq.6.9)
Vrdmax=α_{cw}·**b**w·**z**·**v**1·**f**cd/(cotθ+tanθ), **V**sd/**V**rdmax=0.11, θ=21.8° cotθ=2.50 tanθ=0.40
α_{cw}=1.00 **z**=0.9d, **f**ck=25.0<=60Mpa **v**1=0.60, **f**cd=16.67Mpa
Vrdmax=0.001x1.00x250x0.9x461x0.60x16.67/2.90=357.7 kN

Minimum links for shear reinforcement (EC2 §9.2.2)
Minimum shear reinforcement ratio ρ_{w,min} (EC2 Eq.9.5N)
ρ_{w,min}=(0.08x(**f**ck)^{0.5})/**f**yk, **f**ck=25N/mm², **f**yk=500N/mm², ρ_{w,min}=0.0008
min **A**sw/**s**=10x0.0008x250xsin(90°)= 2.00cm²/m

Maximum longitudinal spacing of links **s**lmax=0.75d(1+cot90°)=345mm (EC2 §9.2.2.6, Eq.9.6N)
Maximum transverse spacing of link legs **s**tmax=0.75d (<=600mm)=345mm (EC2 §9.2.2.8, Eq.9.8N)

Minimum shear reinforcement stirrups Ø8/34.5 (**A**sw/**s**= 2.92cm²/m)

Shear reinforcement: stirrups Ø8/34.5 (A_{sw}/s= 2.92cm²/m)

1.4. Serviceability limit state, crack control

(EC2 EN1992-1-1:2004, §7.3.2, §7.3.3)

Minimum reinforcement areas $A_{s,min}=k_c \cdot k \cdot f_{ct,eff} \cdot A_{ct} / \sigma_s$ (EC2 Eq.7.1)

$b=0.250m, b_{eff}=0.250m, h=0.500m, d=0.461m, N=-40.00kN, \sigma_c=(N/bh)=-0.32N/mm^2, \Phi=14mm$

$\max(h, b_l)=500mm, f_{ctm}=2.60N/mm^2, h_{c,eff}=2.50 \times (h-d)=98mm, k=0.86, k_c=0.47$ (EC2 Eq.7.2)

Min. reinf. without control of crack width, $A_{s,min}=0.47 \times 0.86 \times 2.60 \times 250 \times 98 / 500 = 51mm^2 = 0.51cm^2$

Crack control for crack width $w_k=0.3mm$, using steel diameter $\Phi=14mm$

$\sigma_s = \sigma \cdot s (f_{ctm}/2.9) [k_c \cdot h_{cr} / 2 (h-d)], \sigma_s=14mm, \Phi^*=10mm, (f_{ctm}=2.60, h_{cr}=250mm)$ (EC2 Eq.7.6N)

Steel bar diameter $\Phi^*=10mm$, crack width $w_k=0.3mm$, steel stress $\sigma_s=320N/mm^2$ (EC2 Table 7.2N)

Min. reinforcement for $w_k=0.3mm$ and $\Phi=14mm, A_{s,min}=0.47 \times 0.86 \times 2.60 \times 250 \times 98 / 320 = 80mm^2 = 0.80cm^2$

2. BEAM-002

Design of beam section for bending, shear and axial force

(EC2 EN1992-1-1:2004, EC0 EN1990-1-1:2002)

$b \times h = 0.200 \times 0.600 m, M_{sd} = 100.00 kNm,$

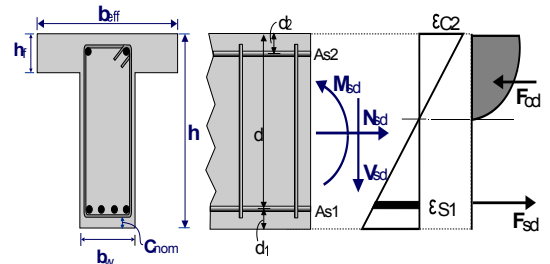
$b_{eff} = 0.300 m, h_f = 0.150 m$

$V_{sd} = 10.00 kN, N_{sd} = 10.00 kN$

Concrete-Steel class: C25/30-S500 (EC2 §3)

Concrete cover : $C_{nom} = 20 mm$ (EC2 §4.4.1)

$\gamma_c = 1.50, \gamma_s = 1.15$ (EC2 Table 2.1N)



2.1. Dimensions and loads

Beam web width $b_w = 0.200 m$, beam height $h = 0.600 m$

Effective flange width $b_{eff} = 0.300 m$, slab thickness $h_f = 0.150 m$

Bending moment $M_{sd} = 100.00 kNm$, Shear $V_{sd} = 10.00 kN$, Axial force $N_{sd} = 10.00 kN$ (tension)

Effective depth of cross section $d_1 = C_{nom} + \Phi_s + 1.1\Phi = 20 + 8 + 1.1 \times 8 = 37mm, d_2 = 37mm, d = 600 - 37 = 563mm$

2.2. Ultimate limit state, design for bending with axial force

(EC2 §6.1, §9.2.1)

Reinforcement for bending with axial force (only tension reinforcement is needed)

Dimensioning for bending: Allgower, G.-Avak, R. Bemessungstabellen nach Eurocode 2 für Rechteck und Plattenbalkenquerschnitte, In: Beton - und Stahlbetonbau 87 (1992)

$M_{sd} = 100kNm, N_{sd} = 10kN, b_{eff} = 300mm, d = 563mm, K_d = 3.13 \times d = 0.11 \epsilon_c / \epsilon_{s2.3/20.0} k_s = 2.40, A_{s1} = 4.37cm^2$

$x = 0.11 \times 563 = 62 < h_f = 150mm$ neutral axis within the depth of top flange

Minimum longitudinal tension reinf., $A_s > 0.26bd \cdot F_{ctm} / f_{yk}$, ($A_{s,min} = 1.52cm^2$) (EC2 §9.2.1.1.1)

Maximum tension or compression reinf., $A_s \leq 0.04A_c$, ($A_{s,max} = 48.00cm^2$) (EC2 §9.2.1.1.3)

Longitudinal reinforcement: 4Ø12 (4.52cm²) (bottom)

2.3. Design against shear failure

(EC2 EN1992-1-1:2004, §6.2, §9.2.2)

Shear capacity without shear reinforcement V_{rdc} (EC2 §6.2.2)

$V_{rdc} = [C_{rdc} \cdot k \cdot (100\rho_1 \cdot f_{ck})^{(0.333)} + k_1 \cdot \sigma_{cp}] \cdot b_w \cdot d$ (EC2 Eq.6.2.a)

$V_{rdc} \geq (v_{min} + k_1 \cdot \sigma_{cp}) \cdot b_w \cdot d$ (EC2 Eq.6.2.b)

$C_{rdc} = 0.18 / \gamma_c = 0.18 / 1.50 = 0.120, f_{ck} = 25.00MPa$

$k = 1 + (200/d)^{1/2} \leq 2, k = 1.60, k_1 = 0.15$

$\rho_1 = A_{s1} / (b_w \cdot d) = 452 / (200 \times 563) = 0.0040$

$\sigma_{cp} = N_{sd} / A_c = -1000 \times 10.00 / 135000 = -0.07N/mm^2$

$v_{min} = 0.035 \cdot k^{(1.50)} \cdot f_{ck}^{1/2} = 0.35N/mm^2$ (EC2 Eq.6.3N)

$V_{rd,c(min)} = 0.001 \times (0.35 - 0.15 \times 0.07) \times 200 \times 563 = 38.24kN$

$V_{rdc} = 0.001 \times [0.120 \times 1.60 \times (0.40 \times 25.00)^{(0.333)} - 0.15 \times 0.07] \times 200 \times 563 = 45.41kN$

$V_{sd} = 10.00 kN \leq V_{rdc} = 45.41 kN, V_{sd} \leq V_{rdc}$ shear reinforcement is not needed

Concrete strut capacity V_{rdmax} (EC2 §6.2.3 Eq.6.9)

$V_{rdmax} = \alpha_{cw} \cdot b_w \cdot z \cdot v_1 \cdot f_{cd} / (\cot\theta + \tan\theta), V_{sd} / V_{rdmax} = 0.03, \theta = 21.8^\circ \cot\theta = 2.50 \tan\theta = 0.40$

$\alpha_{cw} = 1.00 \ z = 0.9d, f_{ck} = 25.0 \leq 60MPa \ v_1 = 0.60, f_{cd} = 16.67Mpa$

$V_{rdmax} = 0.001 \times 1.00 \times 200 \times 0.9 \times 563 \times 0.60 \times 16.67 / 2.90 = 349.6 kN$

Minimum links for shear reinforcement (EC2 §9.2.2)
 Minimum shear reinforcement ratio $\rho_{w,min}$ (EC2 Eq.9.5N)
 $\rho_{w,min} = (0.08 \times (f_{ck})^{0.5}) / f_{yk}$, $f_{ck} = 25 \text{ N/mm}^2$, $f_{yk} = 500 \text{ N/mm}^2$, $\rho_{w,min} = 0.0008$
 $\min A_{sw}/s = 10 \times 0.0008 \times 200 \times \sin(90^\circ) = 1.60 \text{ cm}^2/\text{m}$

Maximum longitudinal spacing of links $s_{lmax} = 0.75d(1 + \cot 90^\circ) = 420 \text{ mm}$ (EC2 §9.2.2.6, Eq.9.6N)
 Maximum transverse spacing of link legs $s_{tmax} = 0.75d (\leq 600 \text{ mm}) = 420 \text{ mm}$ (EC2 §9.2.2.8, Eq.9.8N)

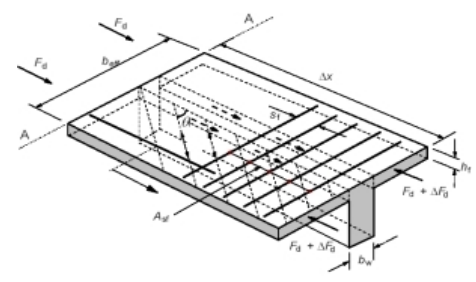
Minimum shear reinforcement stirrups $\emptyset 8/42.0$ ($A_{sw}/s = 2.40 \text{ cm}^2/\text{m}$)

Shear reinforcement: stirrups $\emptyset 8/42.0$ ($A_{sw}/s = 2.40 \text{ cm}^2/\text{m}$)

2.4. Shear between web and flanges

(EC2 EN1992-1-1:2004, §6.2.4)

$F_c = F_s = 0.001 \times 437 \times 435 = 190 \text{ kN}$
 $\Delta F_d = F_c \cdot (b_{eff} - b_w) / (b_{eff}) = 190.0 \times (300 - 200) / (300) = 63 \text{ kN}$
 Beam span $L = 12.40 \text{ m}$, $\Delta x = 0.85 \times 12.40 / 2 = 5.27 \text{ m}$ (EC2 §5.3.2.1)
 $V_{rdmax} = v \cdot h_f \cdot f_{cd} \cdot \sin \theta \cdot \cos \theta$, $f_{cd} = 16.67 \text{ Mpa}$, $\theta = 26.5^\circ$
 $v = 0.6(1 - f_{ck}/250) = 0.54$ (EC2 Eq.6.6N)
 $V_{rdmax} = 0.54 \times 150 \times 16.67 \times \sin 26.5^\circ \times \cos 26.5^\circ = 539 \text{ kN/m}$ (EC2 Eq.6.22)
 $\Delta F_d / \Delta x = 63 / 5.27 = 12 < V_{rdmax} = 539 \text{ kN/m}$, the check is verified
 Transverse reinforcement per unit length A_{sf}/s_f (EC2 Eq.6.21)
 $A_{sf}/s_f = 10 \times 12 / (435 \times \cot 26.5^\circ) = 0.14 \text{ cm}^2/\text{m}$
Transverse reinforcement $A_{sf}/s_f = \emptyset 8/42.0$ ($1.20 \text{ cm}^2/\text{m}$)
 $\Delta F_d / \Delta x = 12 < 0.40 \cdot h_f \cdot F_{ctd} = 0.40 \times 150 \times 1.20 = 72 \text{ kN/m}$
 In case of transverse flexural reinforcement from plate bending,
 No extra reinforcement is needed (EC2 §6.2.4.6)



2.5. Serviceability limit state, crack control

(EC2 EN1992-1-1:2004, §7.3.2, §7.3.3)

Minimum reinforcement areas $A_{s,min} = k_c \cdot k \cdot f_{ct,eff} \cdot A_{ct} / \sigma_s$ (EC2 Eq.7.1)
 $b = 0.200 \text{ m}$, $b_{eff} = 0.300 \text{ m}$, $h = 0.600 \text{ m}$, $d = 0.563 \text{ m}$, $N = -10.00 \text{ kN}$, $\sigma_c = (N/bh) = -0.08 \text{ N/mm}^2$, $\Phi = 12 \text{ mm}$
 $\max(h, b_l) = 600 \text{ mm}$, $f_{ctm} = 2.60 \text{ N/mm}^2$, $h_{c,eff} = 2.50 \times (h - d) = 92 \text{ mm}$, $k = 0.79$, $k_c = 0.42$ (EC2 Eq.7.2)
 Min. reinf. without control of crack width, $A_{s,min} = 0.42 \times 0.79 \times 2.60 \times 200 \times 92 / 500 = 32 \text{ mm}^2 = 0.32 \text{ cm}^2$
 Crack control for crack width $w_k = 0.3 \text{ mm}$, using steel diameter $\emptyset = 12 \text{ mm}$
 $\emptyset_s = \emptyset \cdot s (f_{ctm} / 2.9) [k_c \cdot h_{cr} / 2 (h - d)]$, $\emptyset_s = 12 \text{ mm}$, $\emptyset^* = 8 \text{ mm}$, ($f_{ctm} = 2.60$, $h_{cr} = 300 \text{ mm}$) (EC2 Eq.7.6N)
 Steel bar diameter $\Phi^* = 8 \text{ mm}$, crack width $w_k = 0.3 \text{ mm}$, steel stress $\sigma_s = 360 \text{ N/mm}^2$ (EC2 Table 7.2N)
 Min. reinforcement for $w_k = 0.3 \text{ mm}$ and $\emptyset = 12 \text{ mm}$, $A_{s,min} = 0.42 \times 0.79 \times 2.60 \times 200 \times 92 / 360 = 44 \text{ mm}^2 = 0.44 \text{ cm}^2$

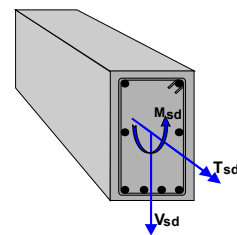
3. BEAM-003

Design of beam section for torsion, bending and shear

(EC2 EN1992-1-1:2004, EC0 EN1990-1-1:2002)

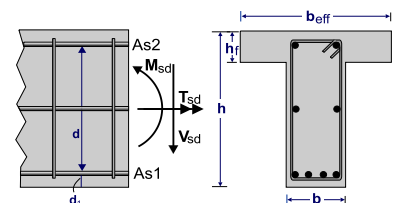
$b \times h = 0.200 \times 0.500 \text{ m}$, $T_{sd} = 10.00 \text{ kNm}$,
 $b_{eff} = 0.800 \text{ m}$, $h_f = 0.180 \text{ m}$
 $M_{sd} = 100.00 \text{ kNm}$, $V_{sd} = 10.00 \text{ kN}$

Concrete-Steel class: C25/30-S500 (EC2 §3)
 Concrete cover : $C_{nom} = 20 \text{ mm}$ (EC2 §4.4.1)
 $\gamma_c = 1.50$, $\gamma_s = 1.15$ (EC2 Table 2.1N)



3.1. Dimensions and loads

Beam web width $b_w = 0.200 \text{ m}$, beam height $h = 0.500 \text{ m}$
 Effective flange width $b_{eff} = 0.800 \text{ m}$, slab thickness $h_f = 0.180 \text{ m}$
 Torsional moment $T_{sd} = 10.00 \text{ kNm}$
 Bending moment $M_{sd} = 100.00 \text{ kNm}$
 Shear force $V_{sd} = 10.00 \text{ kN}$



Effective depth of cross section $d_1 = C_{nom} + \emptyset_s + 0.5\emptyset = 20 + 8 + 0.5 \times 14 = 35 \text{ mm}$, $d_2 = 35 \text{ mm}$, $d = 500 - 35 = 465 \text{ mm}$

3.2. Ultimate limit state, design for bending

(EC2 EN1992-1-1:2004, §6.1, §9.2.1)

Reinforcement for bending (only tension reinforcement is needed)

Dimensioning for bending: Allgower, G.-Avak, R. Bemessungstabellen nach Eurocode 2 für Rechteck und Plattenbalkenquerschnitte, In: Beton - und Stahlbetonbau 87 (1992)

Msd=100.00kNm, beff=800mm, d=465mm, Kd=4.16 $x/d=0.07$ $\epsilon_c/\epsilon_s=1.6/20.0$ $k_s=2.36$, **As1= 5.08cm²**

$x=0.07 \times 465 = 33 < h_f = 180$ mm neutral axis within the depth of top flange

Minimum longitudinal tension reinf., $A_s \geq 0.26bd \cdot F_{ctm}/f_{yk}$, ($A_{s,min} = 1.26\text{cm}^2$) (EC2 §9.2.1.1.1)

Maximum tension or compression reinf., $A_s \leq 0.04A_c$, ($A_{s,max} = 40.00\text{cm}^2$) (EC2 §9.2.1.1.3)

Reinforcement for bending: 3Ø12+1Ø16 (5.40cm²) (bottom)

3.3. Design for shear and torsion

(EC2 EN1992-1-1:2004, §6.3.2.4)

The design torsional resistance moment Trd,max is based on a truss model,

with angle of inclined compression struts at $\theta=40.0^\circ$ ($1.0 < \cot 40.0^\circ = 1.19 < 2.5$) (EC2 Eq.6.7N)

Concrete strut capacity $V_{rd,max}$ (EC2 §6.2.3 Eq.6.9)

$V_{rd,max} = \alpha_{cw} \cdot b_w \cdot z \cdot v_1 \cdot f_{cd} / (\cot \theta + \tan \theta)$, $V_{sd}/V_{rd,max} = 0.02$, $\theta = 40.0^\circ$ $\cot \theta = 1.19$ $\tan \theta = 0.84$

$\alpha_{cw} = 1.00$ $z = 0.9d$, $f_{ck} = 25.0 \leq 60$ Mpa $v_1 = 0.60$, $f_{cd} = 16.67$ Mpa

$V_{rd,max} = 0.001 \times 1.00 \times 200 \times 0.9 \times 465 \times 0.60 \times 16.67 / 2.03 = 412.4$ kN

Torsional resistance moment (EC2 §6.3.2.4)

$Trd,max = 2v \cdot \alpha_{cw} \cdot f_{cd} \cdot A_k \cdot t_{ef} \cdot \sin \theta \cdot \cos \theta$, $\theta = 40.0^\circ$ (EC2 Eq.6.30)

$\alpha_{cw} = 1.00$, $v = 0.6(1 - f_{ck}/250) = 0.6(1 - 25.00/250) = 0.540$ (EC2 Eq.6.9 Eq.6.6N)

$t_{ef} = A/u = 0.500 \times 0.200 / (2 \times 0.500 + 2 \times 0.200) = 0.071\text{m} = 71\text{mm} > 2x_{d1} = 2 \times 35 = 70\text{mm}$

$A_k = (0.500 - 0.071) \times (0.200 - 0.071) = 0.055\text{m}^2 = 55102\text{mm}^2$, $u_k = 2 \times (0.129 + 0.429) = 1.114\text{m} = 1114\text{mm}$

$Trd,max = 2 \times 0.540 \times 1.00 \times 0.001 \times 16.67 \times 55102 \times 0.071 \times 0.643 \times 0.766 = 34.89\text{kNm}$

Shear force and torsion $(T_{sd}/Trd,Max) + (V_{sd}/V_{rd,Max}) \leq 1$ (EC2 EN1992-1-1:2004, §6.3.2)

$(10.00/34.89) + (10.00/412.40) = 0.31 \leq 1$

Shear reinforcement of vertical stirrups (EC2 §6.2.3 Eq.6.8)

$V_{rds} = (A_{sw}/s) \cdot z \cdot f_{ywd} \cdot \cot \theta$, $V_{rds} = 10.00\text{kN}$, $z = 0.9d$, $f_{ywd} = 0.8f_{yk} = 400.00\text{N/mm}^2$, $\cot \theta = 1.19$

$A_{sw}/s = V_{rds} / (z \cdot f_{ywd} \cdot \cot 40.00^\circ) = (1.0\text{E}+006) \times 10.00 / (0.9 \times 465 \times 400 \times 1.19) = 50\text{mm}^2/\text{m}$ ($A_{sw}/s = 0.50\text{cm}^2/\text{m}$)

Required shear reinforcement: ($A_{sw}/s = 0.50\text{cm}^2/\text{m}$)

Required longitudinal reinforcement for torsion (EC2 Eq.6.28)

$A_{sl}/u_k = T_{sd} \cdot \cot 40.0^\circ / (2A_k \cdot f_{yd}) = (1.0\text{E}+007) \times 10.00 \times 1.192 / (2 \times 55102 \times 435) = 2.49\text{cm}^2/\text{m}$

Required torsion links (EC2 Eq.6.26, Eq.6.27, Eq.6.8)

$A_{sw}/s = T_{sd} \cdot \tan 40.0^\circ / (2A_k \cdot f_{yd}) = (1.0\text{E}+007) \times 10.00 \times 0.839 / (2 \times 55102 \times 435) = 1.75\text{cm}^2/\text{m}$

Minimum links for shear reinforcement (EC2 EN1992-1-1:2004, §9.2.2)

Minimum shear reinforcement ratio $\rho_{w,min}$ (EC2 Eq.9.5N)

$\rho_{w,min} = (0.08 \times (f_{ck})^{0.5}) / f_{yk}$, $f_{ck} = 25\text{N/mm}^2$, $f_{yk} = 500\text{N/mm}^2$, $\rho_{w,min} = 0.0008$

$\min A_{sw}/s = 10 \times 0.0008 \times 200 \times \sin(90^\circ) = 1.60\text{cm}^2/\text{m}$

Maximum longitudinal spacing of links $s_{l,max} = 0.75d(1 + \cot 90^\circ) = 345\text{mm}$ (EC2 §9.2.2.6, Eq.9.6N)

maximum space between torsion links $s_w = 135\text{mm}$ ($\leq \min(u/8 = 1114/8, 200)$) (EC2 §9.2.3.3)

maximum spacing of longitudinal bars for torsion 130mm ($\leq 350\text{mm}$) (EC2 §9.2.3.4)

Maximum transverse spacing of link legs $s_{t,max} = 0.75d$ ($\leq 600\text{mm}$) = 345mm (EC2 §9.2.2.8, Eq.9.8N)

Torsion-Shear reinforcement: closed links Ø8/13.5 ($A_{sw}/s = 7.45\text{cm}^2/\text{m}$)

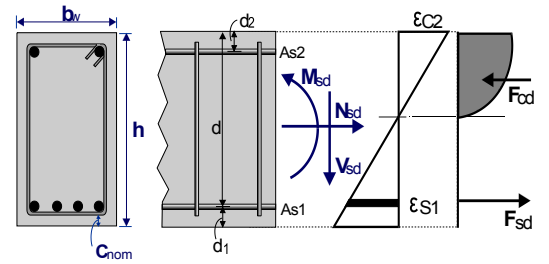
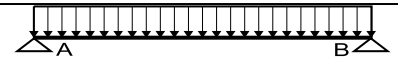
Longitudinal reinforcement for torsion: Ø10/13.0 (8Ø10) (6.04cm²/m)

4. BEAM-004

One span beam in composite loading

(EC2 EN1992-1-1:2004, EC0 EN1990-1-1:2002)

Concrete-Steel class: C25/30-S500 (EC2 §3)
 Concrete cover : C_{nom}=20 mm (EC2 §4.4.1)
 Concrete weight : 25.0 kN/m³
 γ_c=1.50, γ_s=1.15 (EC2 Table 2.1N)



4.1. Dimensions and loads

Beam (rectangular section) , span L=6.000 m
 L=6.000m, bw=0.250m, h=0.500m

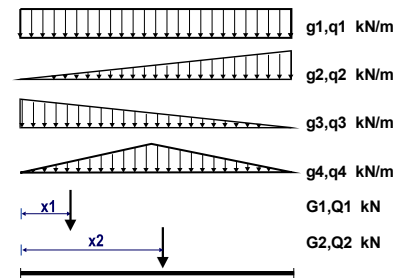
Partial safety factors for actions : γ_G=1.35, γ_Q=1.50 (EC0 Annex A1)

Combination of variable actions : ψ₁=0.60, ψ₂=0.30

Effective depth of cross section d=h-d₁, d₁=C_{nom}c+Øs+0.5Ø=20+8+0.5×14=35mm

Beam loads

beam self weight go= 3.13 kN/m
 uniform load g1= 29.00 kN/m q1= 10.00 kN/m
 triangular load g2= 0.00 kN/m q2= 0.00 kN/m
 triangular load g3= 0.00 kN/m q3= 0.00 kN/m
 triangular load g4= 0.00 kN/m q4= 0.00 kN/m
 concentrated load G1= 0.00 kN Q1= 0.00 kN x1= 0.000 m
 concentrated load G2= 0.00 kN Q2= 0.00 kN x2= 0.000 m



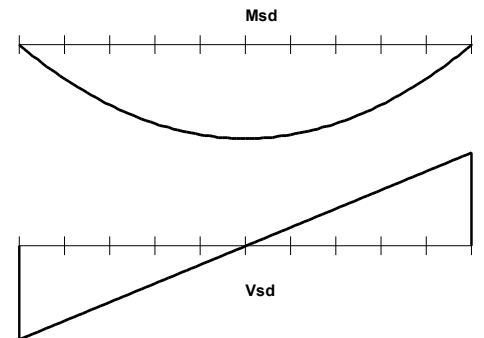
Cross section values (area A, moment of inertia I_{xx}, centroid yc)

Span-1 L= 6.000m, A=0.12500m² (1.25E+005mm²), I_{xx}=0.002260m⁴ (2.26E+009mm⁴), yc=0.000m (0mm)

4.2. Design actions, shearing forces and bending moments

Bending moments and shears, load combination 1.35g+1.50q

x/L=0.00, x= 0.00m, Msd= 0.00 kNm, Vsd= 175.11 kN
x/L=0.10, x= 0.60m, Msd= 94.56 kNm, Vsd= 140.09 kN
x/L=0.20, x= 1.20m, Msd= 168.10 kNm, Vsd= 105.06 kN
x/L=0.30, x= 1.80m, Msd= 220.63 kNm, Vsd= 70.04 kN
x/L=0.40, x= 2.40m, Msd= 252.15 kNm, Vsd= 35.02 kN
x/L=0.50, x= 3.00m, Msd= 262.66 kNm, Vsd= 0.00 kN
x/L=0.60, x= 3.60m, Msd= 252.15 kNm, Vsd= -35.02 kN
x/L=0.70, x= 4.20m, Msd= 220.63 kNm, Vsd= -70.04 kN
x/L=0.80, x= 4.80m, Msd= 168.10 kNm, Vsd= -105.06 kN
x/L=0.90, x= 5.40m, Msd= 94.56 kNm, Vsd= -140.08 kN
x/L=1.00, x= 6.00m, Msd= 0.00 kNm, Vsd= -175.11 kN



VsdA= 175.11 kN, VsdB= 175.11 kN, maxMsd= 262.66 kNm, maxVsd= 175.11 kN

Maximum span moment Msd=262.66 kNm (x=3.000m)

Maximum shear forces at distance d from support face

Span-A, b/2+d=0.557m, VsdA= 143.59kN, VsdB= 143.59kN

4.3. Span Ultimate limit state, design for bending

(EC2 EN1992-1-1:2004, §6.1, §9.2.1)

Effective depth of cross section $d_1 = C_{nomc} + \varnothing_s + 1.1\varnothing = 20 + 8 + 1.1 \times 14 = 43 \text{ mm}$, $d_2 = 43 \text{ mm}$, $d = 500 - 43 = 457 \text{ mm}$

Reinforcement for bending (tension and compression, reinforcement is needed)

 $M_{sd} = 262.66 \text{ kNm}$, $b_w = 250 \text{ mm}$, $d = 457 \text{ mm}$, $K_d = 1.41$, $k_{s1} = 2.78$, $k_{s2} = 0.42$, **$A_{s1} = 16.00$, $A_{s2} = 2.44 \text{ cm}^2$** Minimum longitudinal tension reinf., $A_s > 0.26bd \cdot f_{ctm} / f_{yk}$, ($A_{s, \text{min}} = 1.54 \text{ cm}^2$) (EC2 §9.2.1.1.1)Maximum tension or compression reinf., $A_s \leq 0.04A_c$, ($A_{s, \text{max}} = 50.00 \text{ cm}^2$) (EC2 §9.2.1.1.3)**Reinforcement for bending: 4Ø18+2Ø20 (16.44 cm²) (bottom), 2Ø14 (3.08 cm²) (top)****4.4. Span Design against shear failure**

(EC2 EN1992-1-1:2004, §6.2, §9.2.2)

Shear capacity without shear reinforcement V_{rdc} (EC2 §6.2.2) $V_{rdc} = [C_{rdc} \cdot k \cdot (100\rho_1 \cdot f_{ck})^{0.333} + k_1 \cdot \sigma_{cp}] \cdot b_w \cdot d$ (EC2 Eq.6.2.a) $V_{rdc} \geq (v_{min} + k_1 \cdot \sigma_{cp}) \cdot b_w \cdot d$ (EC2 Eq.6.2.b) $C_{rdc} = 0.18 / \gamma_c = 0.18 / 1.50 = 0.120$, $f_{ck} = 25.00 \text{ MPa}$ $k = 1 + (200/d)^{1/2} \leq 2$, $k = 1.66$, $k_1 = 0.15$ $\rho_1 = A_{s1} / (b_w \cdot d) = 1644 / (250 \times 457) = 0.0144$ $v_{min} = 0.035 \cdot k^{1.50} \cdot f_{ck}^{1/2} = 0.37 \text{ N/mm}^2$ (EC2 Eq.6.3N) $V_{rd, c(\text{min})} = 0.001 \times (0.37) \times 250 \times 457 = 42.24 \text{ kN}$ $V_{rdc} = 0.001 \times [0.120 \times 1.66 \times (1.44 \times 25.00)^{0.333}] \times 250 \times 457 = 75.08 \text{ kN}$ $V_{sd} = 143.59 \text{ kN} > V_{rdc} = 75.08 \text{ kN}$, **$V_{sd} > V_{rdc}$ shear reinforcement is needed**Concrete strut capacity V_{rdmax} (EC2 §6.2.3 Eq.6.9) $V_{rdmax} = \alpha_{cw} \cdot b_w \cdot z \cdot v_1 \cdot f_{cd} / (\cot\theta + \tan\theta)$, $V_{sd} / V_{rdmax} = 0.41$, $\theta = 21.8^\circ$, $\cot\theta = 2.50$, $\tan\theta = 0.40$ $\alpha_{cw} = 1.00$, $z = 0.9d$, $f_{ck} = 25.0 \leq 60 \text{ MPa}$, $v_1 = 0.60$, $f_{cd} = 16.67 \text{ MPa}$ $V_{rdmax} = 0.001 \times 1.00 \times 250 \times 0.9 \times 457 \times 0.60 \times 16.67 / 2.90 = 354.3 \text{ kN}$ $V_{sd} = 143.6 \text{ kN} < 354.3 \text{ kN} = V_{rdmax}$, the check is verifiedShear reinforcement of vertical stirrups (EC2 §6.2.3 Eq.6.8) $V_{rds} = (A_{sw}/s) \cdot z \cdot f_{ywd} \cdot \cot\theta$, $V_{rds} = 143.59 \text{ kN}$, $z = 0.9d$, $f_{ywd} = 0.8f_{yk} = 400.00 \text{ N/mm}^2$, $\cot\theta = 2.50$ $A_{sw}/s = V_{rds} / (z \cdot f_{ywd} \cdot \cot\theta) = (1.0E+006) \times 143.59 / (0.9 \times 457 \times 400 \times 2.50) = 349 \text{ mm}^2/\text{m}$ ($A_{sw}/s = 3.49 \text{ cm}^2/\text{m}$)Required shear reinforcement: ($A_{sw}/s = 3.49 \text{ cm}^2/\text{m}$)Minimum links for shear reinforcement (EC2 §9.2.2)Minimum shear reinforcement ratio $\rho_{w, \text{min}}$ (EC2 Eq.9.5N) $\rho_{w, \text{min}} = (0.08 \times (f_{ck})^{0.5}) / f_{yk}$, $f_{ck} = 25 \text{ N/mm}^2$, $f_{yk} = 500 \text{ N/mm}^2$, $\rho_{w, \text{min}} = 0.0008$ $\text{min } A_{sw}/s = 10 \times 0.0008 \times 250 \times \sin(90^\circ) = 2.00 \text{ cm}^2/\text{m}$ Maximum longitudinal spacing of links $s_{lmax} = 0.75d(1 + \cot 90^\circ) = 340 \text{ mm}$ (EC2 §9.2.2.6, Eq.9.6N)Maximum transverse spacing of link legs $s_{tmax} = 0.75d (\leq 600 \text{ mm}) = 340 \text{ mm}$ (EC2 §9.2.2.8, Eq.9.8N)Minimum shear reinforcement stirrups $\varnothing 8/34.0$ ($A_{sw}/s = 2.96 \text{ cm}^2/\text{m}$)**Span Shear reinforcement: stirrups $\varnothing 8/28.5$** ($A_{sw}/s = 3.53 \text{ cm}^2/\text{m}$)**4.5. Serviceability limit state, crack control**

(EC2 EN1992-1-1:2004, §7.3.2, §7.3.3)

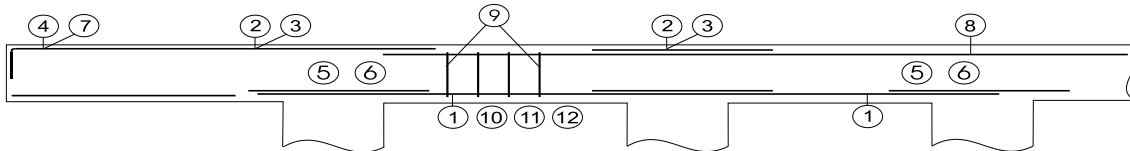
Minimum reinforcement areas $A_{s, \text{min}} = k_c \cdot k \cdot f_{ct, \text{eff}} \cdot A_{ct} / \sigma_s$ (EC2 Eq.7.1) $b = 0.250 \text{ m}$, $b_{eff} = 0.250 \text{ m}$, $h = 0.500 \text{ m}$, $d = 0.457 \text{ m}$, $N = 0.00 \text{ kN}$, $\sigma_c = (N/bh) = 0.00 \text{ N/mm}^2$, $\Phi = 18 \text{ mm}$ $\max(h, b_1) = 500 \text{ mm}$, $f_{ctm} = 2.60 \text{ N/mm}^2$, $h_{c, \text{eff}} = 2.50 \times (h - d) = 108 \text{ mm}$, $k = 0.86$, $k_c = 0.40$ (EC2 Eq.7.2)Min. reinf. without control of crack width, $A_{s, \text{min}} = 0.40 \times 0.86 \times 2.60 \times 250 \times 108 / 500 = 48 \text{ mm}^2 = 0.48 \text{ cm}^2$ Crack control for crack width $w_k = 0.3 \text{ mm}$, using steel diameter $\varnothing = 18 \text{ mm}$ $\varnothing_s = \varnothing^* \cdot s (f_{ctm} / 2.9) [k_c \cdot h_{cr} / 2(h - d)]$, $\varnothing_s = 18 \text{ mm}$, $\varnothing^* = 17 \text{ mm}$, ($f_{ctm} = 2.60$, $h_{cr} = 250 \text{ mm}$) (EC2 Eq.7.6N)Steel bar diameter $\varnothing^* = 17 \text{ mm}$, crack width $w_k = 0.3 \text{ mm}$, steel stress $\sigma_s = 236 \text{ N/mm}^2$ (EC2 Table 7.2N)Min. reinforcement for $w_k = 0.3 \text{ mm}$ and $\varnothing = 18 \text{ mm}$, $A_{s, \text{min}} = 0.40 \times 0.86 \times 2.60 \times 250 \times 108 / 236 = 102 \text{ mm}^2 = 1.02 \text{ cm}^2$ **4.6. Serviceability limit state, deflection control**

(EC2 EN1992-1-1:2004, §7.4.2)

Span/effective depth, must be $L/d \leq \text{limit of EC2 Table 7.4N}$ Span-, $K = 1.00$, $\rho = 1.401\%$, $L/d = 6000 / 457 = 13.14 < 14.59$

4.7. Reinforcing bar schedule

Num		type	reinforcing bar [mm]	items	∅	g/m [kg/m]	length [m]	weight [kg]
Total weight [kg]								0.00

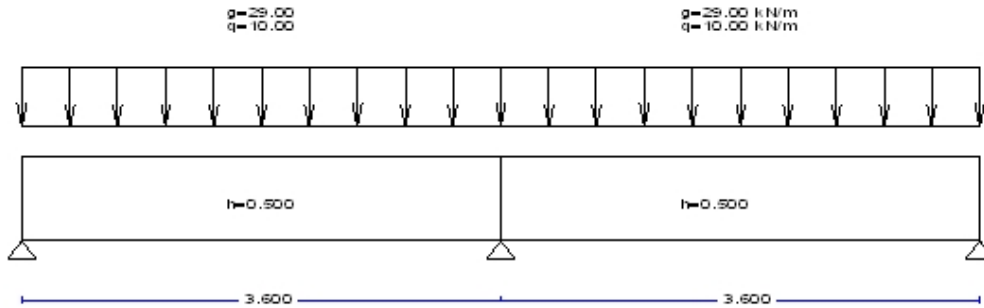


5. BEAM-005

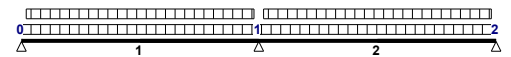
Continuous beam with distributed loads

(EC2 EN1992-1-1:2004, EC0 EN1990-1-1:2002)

C25/30 - S500

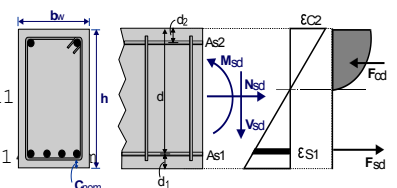


Concrete-Steel class: C25/30-S500 (EC2 §3)
 Concrete cover : C_{nom}=20 mm (EC2 §4.4.1)
 Concrete weight : 25.0 kN/m³
 γ_c=1.50, γ_s=1.15 (EC2 Table 2.1N)



5.1. Dimensions and loads

Continuous beam (rectangular section), number of spans=2
 Partial safety factors for actions : γ_G=1.35, γ_Q=1.50 (EC0 Annex A1)
 Combination of variable actions : ψ₁=0.60, ψ₂=0.30
 Effective depth of cross section d=h-d₁, d₁=C_{nom}+∅_s+0.5∅=20+8+0.5×1



Spans, widths, thickness, load on spans (g=self weight +dead, q=live)

Span-1 L= 3.60m bw=0.250m beff=0.250m h=0.500m g=3.13+ 29.00= 32.13kN/m q= 10.00kN/m
 Span-2 L= 3.60m bw=0.250m beff=0.250m h=0.500m g=3.13+ 29.00= 32.13kN/m q= 10.00kN/m

Cross section values (area A, moment of inertia Ixx, centroid yc)

Span-1 L= 3.600m, A=0.12500m² (1.25E+005mm²), Ixx=0.00260m⁴ (2.60E+009mm⁴), yc=0.000m (0mm)
 Span-2 L= 3.600m, A=0.12500m² (1.25E+005mm²), Ixx=0.00260m⁴ (2.60E+009mm⁴), yc=0.000m (0mm)

5.2. Shearing forces and bending moments

Maximum bending moments at spans for load combinations 1.35g+1.50q

Span-1, Msd= 61.46 kNm, xo=1.451 m, x1=0.000m, x2=0.698m
 Span-2, Msd= 61.46 kNm, xo=2.149 m, x1=0.698m, x2=0.000m

Maximum bending moments at supports for load combinations 1.35g+1.50q

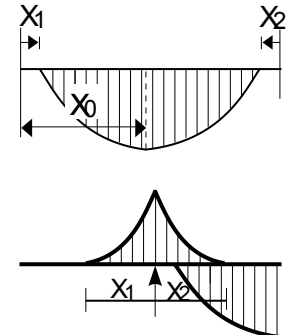
Support-0, Msd= 0.00 kNm, x1=0.000 m, x2=0.000 m
 Support-1, Msd= -94.56 kNm, x1=0.900 m, x2=0.900 m
 Support-2, Msd= 0.00 kNm, x1=0.000 m, x2=0.000 m

Maximum shear forces for load combinations 1.35g+1.50q

Span-1, Vsd,left= 78.80 kN, Vsd,right=-131.33 kN
 Span-2, Vsd,left= 127.95 kN, Vsd,right= -82.17 kN

Maximum reactions due to dead and live loads (Rg and Rq)

Support-0, Rg(x1.35)= 58.55 kN, Rq(x1.50)= 23.63 kN
 Support-1, Rg(x1.35)= 195.16 kN, Rq(x1.50)= 67.50 kN
 Support-2, Rg(x1.35)= 58.55 kN, Rq(x1.50)= 23.63 kN



5.3. Design actions, shearing forces and bending moments

Design action values after moment redistribution by 0%

Reduction of support moments to moments at support faces (bsup=0.20 m)
 Check for minimum values, (0.65ql²/8 or 0.65ql²/12)

(EC2 §5.5)
 (EC2 §5.3.2.2.3)
 (EC2 §5.3.2.2.3N)

Maximum span bending moments and shear forces for load combinations 1.35g+1.50q

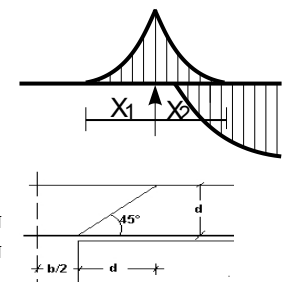
Span-1, Msd= 61.46 kNm, Vsd,left= 84.70 kN, Vsd,right=-125.42 kN
 Span-2, Msd= 61.46 kNm, Vsd,left= 125.42 kN, Vsd,right= -84.70 kN

Maximum bending moments at supports for load combinations 1.35g+1.50q

Support-0, Msd= 0.00 kNm, x1=0.000 m, x2=0.000 m
 Support-1, Msd= -82.01 kNm, x1=0.900 m, x2=0.900 m
 Support-2, Msd= 0.00 kNm, x1=0.000 m, x2=0.000 m

Maximum shear forces at distance d from support face 1.35g+1.50q

Span-1, b/2+d=0.565m, 1.35g+1.50q=58.37kN/m, VsdA= 51.72kN, VsdB= 92.45kN
 Span-2, b/2+d=0.565m, 1.35g+1.50q=58.37kN/m, VsdA= 92.45kN, VsdB= 51.72kN



5.4. Ultimate limit state, design for bending

(EC2 EN1992-1-1:2004, §6.1, §9.2.1)

Span-1

Effective depth of cross section d1=Cnomc+ø_s+0.5ø=20+8+0.5x14=35mm, d2=35mm, d=500-35=465mm

Reinforcement for bending (only tension reinforcement is needed)

Msd= 61.46kNm, bw=250mm, d=465mm, Kd=2.97 x/d=0.11 ε_c/ε_s=2.6/20.0 ks=2.41, **As1= 3.18cm²**

Minimum longitudinal tension reinf., As>=0.26bd·Fctm/fyk, (As,min= 1.57cm²) (EC2 §9.2.1.1.1)

Maximum tension or compression reinf., As<=0.04Ac, (As,max=50.00cm²) (EC2 §9.2.1.1.3)

Reinforcement for bending: 4Ø10 (3.14cm²) (bottom)

Span-2

Effective depth of cross section $d_1 = C_{nomc} + \phi_s + 0.5\phi = 20 + 8 + 0.5 \times 14 = 35\text{mm}$, $d_2 = 35\text{mm}$, $d = 500 - 35 = 465\text{mm}$
 Reinforcement for bending (only tension reinforcement is needed)
 $M_{sd} = 61.46\text{kNm}$, $b_w = 250\text{mm}$, $d = 465\text{mm}$, $K_d = 2.97$ $x/d = 0.11$ $\epsilon_c/\epsilon_s = 2.6/20.0$ $k_s = 2.41$, **$A_{s1} = 3.18\text{cm}^2$**
 Minimum longitudinal tension reinf., $A_s >= 0.26bd \cdot F_{ctm}/f_{yk}$, ($A_{s,min} = 1.57\text{cm}^2$)
 Maximum tension or compression reinf., $A_s <= 0.04A_c$, ($A_{s,max} = 50.00\text{cm}^2$)
Reinforcement for bending: 4Ø10 (3.14cm²) (bottom)

Support-1

Effective depth of cross section $d_1 = C_{nomc} + \phi_s + 0.5\phi = 20 + 8 + 0.5 \times 14 = 35\text{mm}$, $d_2 = 35\text{mm}$, $d = 500 - 35 = 465\text{mm}$
 Reinforcement for bending (only tension reinforcement is needed)
 $M_{sd} = 82.01\text{kNm}$, $b_w = 250\text{mm}$, $d = 465\text{mm}$, $K_d = 2.57$ $x/d = 0.14$ $\epsilon_c/\epsilon_s = 3.3/20.0$ $k_s = 2.44$, **$A_{s2} = 4.31\text{cm}^2$**
 Minimum longitudinal tension reinf., $A_s >= 0.26bd \cdot F_{ctm}/f_{yk}$, ($A_{s,min} = 1.57\text{cm}^2$) (EC2 §9.2.1.1.1)
 Maximum tension or compression reinf., $A_s <= 0.04A_c$, ($A_{s,max} = 50.00\text{cm}^2$) (EC2 §9.2.1.1.3)
Reinforcement for bending: 4Ø10+1Ø12 (4.27cm²) (top)

5.5. Design against shear failure

(EC2 EN1992-1-1:2004, §6.2, §9.2.2)

Span-1 left

Shear capacity without shear reinforcement V_{rdc} (EC2 §6.2.2)
 $V_{rdc} = [C_{rdc} \cdot k \cdot (100\rho_1 \cdot f_{ck})^{(0.333)} + k_1 \cdot \sigma_{cp}] \cdot b_w \cdot d$, $V_{rdc} >= (v_{min} + k_1 \cdot \sigma_{cp}) \cdot b_w \cdot d$ (EC2 Eq.6.2.a,b)
 $C_{rdc} = 0.18/\gamma_c = 0.18/1.50 = 0.120$, $f_{ck} = 25.00\text{MPa}$, $k = 1 + (200/d)^{1/2} \leq 2$, $k = 1.66$, $k_1 = 0.15$
 $V_{rd,c(min)} = 0.001x(0.37) \times 250 \times 465 = 43.01\text{kN}$, $v_{min} = 0.035 \cdot k^{(1.50)} \cdot f_{ck}^{1/2} = 0.37\text{N/mm}^2$ (EC2 Eq.6.3N)
 $\rho_1 = 314/(250 \times 465) = 0.0027$, $V_{rdc} = 0.001x[0.120 \times 1.66x(0.27 \times 25.00)^{(0.333)}] \times 250 \times 465 = 43.76\text{kN}$
 $V_{sd} = 51.72\text{ kN} > V_{rdc} = 43.76\text{ kN}$, **$V_{sd} > V_{rdc}$ shear reinforcement is needed**

Concrete strut capacity V_{rdmax}

(EC2 §6.2.3 Eq.6.9)

$V_{rdmax} = \alpha_{cw} \cdot b_w \cdot z \cdot v_1 \cdot f_{cd} / (\cot\theta + \tan\theta)$, $V_{sd}/V_{rdmax} = 0.14$, $\theta = 21.8^\circ$ $\cot\theta = 2.50$ $\tan\theta = 0.40$
 $\alpha_{cw} = 1.00$ $z = 0.9d$, $f_{ck} = 25.0 \leq 60\text{Mpa}$ $v_1 = 0.60$, $f_{cd} = 16.67\text{Mpa}$
 $V_{rdmax} = 0.001 \times 1.00 \times 250 \times 0.9 \times 465 \times 0.60 \times 16.67 / 2.90 = 360.8\text{ kN}$
 $V_{sd} = 51.7\text{ kN} < 360.8\text{ kN} = V_{rdmax}$, the check is verified

Shear reinforcement of vertical stirrups

(EC2 §6.2.3 Eq.6.8)

$V_{rds} = (A_{sw}/s) \cdot z \cdot f_{ywd} \cdot \cot\theta$, $V_{rds} = 51.72\text{kN}$, $z = 0.9d$, $f_{ywd} = 0.8f_{yk} = 400.00\text{N/mm}^2$, $\cot\theta = 2.50$
 $A_{sw}/s = V_{rds} / (z \cdot f_{ywd} \cdot \cot\theta) = (1.0E+006) \times 51.72 / (0.9 \times 465 \times 400 \times 2.50) = 124\text{mm}^2/\text{m}$ ($A_{sw}/s = 1.24\text{cm}^2/\text{m}$)
 Required shear reinforcement: ($A_{sw}/s = 1.24\text{cm}^2/\text{m}$)

Minimum links for shear reinforcement

(EC2 §9.2.2)

Minimum shear reinforcement ratio $\rho_{w,min}$ (EC2 Eq.9.5N)
 $\rho_{w,min} = (0.08 \times (f_{ck})^{(0.5)}) / f_{yk}$, $f_{ck} = 25\text{N/mm}^2$, $f_{yk} = 500\text{N/mm}^2$, $\rho_{w,min} = 0.0008$
 $\min A_{sw}/s = 10 \times 0.0008 \times 250 \times \sin(90^\circ) = 2.00\text{cm}^2/\text{m}$
 Maximum longitudinal spacing of links $s_{lmax} = 0.75d(1 + \cot 90^\circ) = 345\text{mm}$ (EC2 §9.2.2.6, Eq.9.6N)
 Maximum transverse spacing of link legs $s_{tmax} = 0.75d (\leq 600\text{mm}) = 345\text{mm}$ (EC2 §9.2.2.8, Eq.9.8N)
 Minimum shear reinforcement stirrups $\phi_8/34.5$ ($A_{sw}/s = 2.92\text{cm}^2/\text{m}$)

Span-1 left Shear reinforcement: stirrups Ø8/34.5 ($A_{sw}/s = 2.92\text{cm}^2/\text{m}$)**Span-1 right**

Shear capacity without shear reinforcement V_{rdc} (EC2 §6.2.2)
 $V_{rdc} = [C_{rdc} \cdot k \cdot (100\rho_1 \cdot f_{ck})^{(0.333)} + k_1 \cdot \sigma_{cp}] \cdot b_w \cdot d$, $V_{rdc} >= (v_{min} + k_1 \cdot \sigma_{cp}) \cdot b_w \cdot d$ (EC2 Eq.6.2.a,b)
 $C_{rdc} = 0.18/\gamma_c = 0.18/1.50 = 0.120$, $f_{ck} = 25.00\text{MPa}$, $k = 1 + (200/d)^{1/2} \leq 2$, $k = 1.66$, $k_1 = 0.15$
 $V_{rd,c(min)} = 0.001x(0.37) \times 250 \times 465 = 43.01\text{kN}$, $v_{min} = 0.035 \cdot k^{(1.50)} \cdot f_{ck}^{1/2} = 0.37\text{N/mm}^2$ (EC2 Eq.6.3N)
 $\rho_1 = 314/(250 \times 465) = 0.0027$, $V_{rdc} = 0.001x[0.120 \times 1.66x(0.27 \times 25.00)^{(0.333)}] \times 250 \times 465 = 43.76\text{kN}$
 $V_{sd} = 92.45\text{ kN} > V_{rdc} = 43.76\text{ kN}$, **$V_{sd} > V_{rdc}$ shear reinforcement is needed**

Concrete strut capacity V_{rdmax}

(EC2 §6.2.3 Eq.6.9)

$V_{rdmax} = \alpha_{cw} \cdot b_w \cdot z \cdot v_1 \cdot f_{cd} / (\cot\theta + \tan\theta)$, $V_{sd}/V_{rdmax} = 0.26$, $\theta = 21.8^\circ$ $\cot\theta = 2.50$ $\tan\theta = 0.40$
 $\alpha_{cw} = 1.00$ $z = 0.9d$, $f_{ck} = 25.0 \leq 60\text{Mpa}$ $v_1 = 0.60$, $f_{cd} = 16.67\text{Mpa}$
 $V_{rdmax} = 0.001 \times 1.00 \times 250 \times 0.9 \times 465 \times 0.60 \times 16.67 / 2.90 = 360.8\text{ kN}$
 $V_{sd} = 92.4\text{ kN} < 360.8\text{ kN} = V_{rdmax}$, the check is verified

Shear reinforcement of vertical stirrups (EC2 §6.2.3 Eq.6.8)

$V_{rds} = (A_{sw}/s) z \cdot f_{ywd} \cdot \cot \theta$, $V_{rds} = 92.45 \text{ kN}$, $z = 0.9d$, $f_{ywd} = 0.8 f_{yk} = 400.00 \text{ N/mm}^2$, $\cot \theta = 2.50$
 $A_{sw}/s = V_{rds} / (z \cdot f_{ywd} \cdot \cot 21.80^\circ) = (1.0 \text{E} + 006) \times 92.45 / (0.9 \times 465 \times 400 \times 2.50) = 221 \text{ mm}^2/\text{m}$ ($A_{sw}/s = 2.21 \text{ cm}^2/\text{m}$)
 Required shear reinforcement: ($A_{sw}/s = 2.21 \text{ cm}^2/\text{m}$)

Minimum links for shear reinforcement (EC2 §9.2.2)

Minimum shear reinforcement ratio $\rho_{w,min}$ (EC2 Eq.9.5N)
 $\rho_{w,min} = (0.08 \times (f_{ck})^{0.5}) / f_{yk}$, $f_{ck} = 25 \text{ N/mm}^2$, $f_{yk} = 500 \text{ N/mm}^2$, $\rho_{w,min} = 0.0008$
 $\min A_{sw}/s = 10 \times 0.0008 \times 250 \times \sin(90^\circ) = 2.00 \text{ cm}^2/\text{m}$
 Maximum longitudinal spacing of links $s_{lmax} = 0.75d(1 + \cot 90^\circ) = 345 \text{ mm}$ (EC2 §9.2.2.6, Eq.9.6N)
 Maximum transverse spacing of link legs $s_{tmax} = 0.75d$ ($\leq 600 \text{ mm}$) = 345 mm (EC2 §9.2.2.8, Eq.9.8N)
 Minimum shear reinforcement stirrups $\emptyset 8/34.5$ ($A_{sw}/s = 2.92 \text{ cm}^2/\text{m}$)

Span-1 right Shear reinforcement: stirrups $\emptyset 8/34.5$ ($A_{sw}/s = 2.92 \text{ cm}^2/\text{m}$)

Span-2 left

Shear capacity without shear reinforcement V_{rdc} (EC2 §6.2.2)

$V_{rdc} = [C_{rdc} \cdot k \cdot (100 \rho_1 \cdot f_{ck})^{0.333} + k_1 \cdot \sigma_{cp}] \cdot b_w \cdot d$, $V_{rdc} \geq (v_{min} + k_1 \cdot \sigma_{cp}) \cdot b_w \cdot d$ (EC2 Eq.6.2.a,b)
 $C_{rdc} = 0.18 / \gamma_c = 0.18 / 1.50 = 0.120$, $f_{ck} = 25.00 \text{ MPa}$, $k = 1 + (200/d)^{1/2} \leq 2$, $k = 1.66$, $k_1 = 0.15$
 $V_{rd,c(min)} = 0.001 \times (0.37) \times 250 \times 465 = 43.01 \text{ kN}$, $v_{min} = 0.035 \cdot k^{1.50} \cdot f_{ck}^{1/2} = 0.37 \text{ N/mm}^2$ (EC2 Eq.6.3N)
 $\rho_1 = 314 / (250 \times 465) = 0.0027$, $V_{rdc} = 0.001 \times [0.120 \times 1.66 \times (0.27 \times 25.00)^{0.333}] \times 250 \times 465 = 43.76 \text{ kN}$
 $V_{sd} = 92.45 \text{ kN} > V_{rdc} = 43.76 \text{ kN}$, **Vsd > Vrdc shear reinforcement is needed**

Concrete strut capacity V_{rdmax} (EC2 §6.2.3 Eq.6.9)

$V_{rdmax} = \alpha_{cw} \cdot b_w \cdot z \cdot v_1 \cdot f_{cd} / (\cot \theta + \tan \theta)$, $V_{sd} / V_{rdmax} = 0.26$, $\theta = 21.8^\circ$, $\cot \theta = 2.50$, $\tan \theta = 0.40$
 $\alpha_{cw} = 1.00$, $z = 0.9d$, $f_{ck} = 25.0 \leq 60 \text{ MPa}$, $v_1 = 0.60$, $f_{cd} = 16.67 \text{ MPa}$
 $V_{rdmax} = 0.001 \times 1.00 \times 250 \times 0.9 \times 465 \times 0.60 \times 16.67 / 2.90 = 360.8 \text{ kN}$
 $V_{sd} = 92.4 \text{ kN} < 360.8 \text{ kN} = V_{rdmax}$, the check is verified

Shear reinforcement of vertical stirrups (EC2 §6.2.3 Eq.6.8)

$V_{rds} = (A_{sw}/s) z \cdot f_{ywd} \cdot \cot \theta$, $V_{rds} = 92.45 \text{ kN}$, $z = 0.9d$, $f_{ywd} = 0.8 f_{yk} = 400.00 \text{ N/mm}^2$, $\cot \theta = 2.50$
 $A_{sw}/s = V_{rds} / (z \cdot f_{ywd} \cdot \cot 21.80^\circ) = (1.0 \text{E} + 006) \times 92.45 / (0.9 \times 465 \times 400 \times 2.50) = 221 \text{ mm}^2/\text{m}$ ($A_{sw}/s = 2.21 \text{ cm}^2/\text{m}$)
 Required shear reinforcement: ($A_{sw}/s = 2.21 \text{ cm}^2/\text{m}$)

Minimum links for shear reinforcement (EC2 §9.2.2)

Minimum shear reinforcement ratio $\rho_{w,min}$ (EC2 Eq.9.5N)
 $\rho_{w,min} = (0.08 \times (f_{ck})^{0.5}) / f_{yk}$, $f_{ck} = 25 \text{ N/mm}^2$, $f_{yk} = 500 \text{ N/mm}^2$, $\rho_{w,min} = 0.0008$
 $\min A_{sw}/s = 10 \times 0.0008 \times 250 \times \sin(90^\circ) = 2.00 \text{ cm}^2/\text{m}$
 Maximum longitudinal spacing of links $s_{lmax} = 0.75d(1 + \cot 90^\circ) = 345 \text{ mm}$ (EC2 §9.2.2.6, Eq.9.6N)
 Maximum transverse spacing of link legs $s_{tmax} = 0.75d$ ($\leq 600 \text{ mm}$) = 345 mm (EC2 §9.2.2.8, Eq.9.8N)
 Minimum shear reinforcement stirrups $\emptyset 8/34.5$ ($A_{sw}/s = 2.92 \text{ cm}^2/\text{m}$)

Span-2 left Shear reinforcement: stirrups $\emptyset 8/34.5$ ($A_{sw}/s = 2.92 \text{ cm}^2/\text{m}$)

Span-2 right

Shear capacity without shear reinforcement V_{rdc} (EC2 §6.2.2)

$V_{rdc} = [C_{rdc} \cdot k \cdot (100 \rho_1 \cdot f_{ck})^{0.333} + k_1 \cdot \sigma_{cp}] \cdot b_w \cdot d$, $V_{rdc} \geq (v_{min} + k_1 \cdot \sigma_{cp}) \cdot b_w \cdot d$ (EC2 Eq.6.2.a,b)
 $C_{rdc} = 0.18 / \gamma_c = 0.18 / 1.50 = 0.120$, $f_{ck} = 25.00 \text{ MPa}$, $k = 1 + (200/d)^{1/2} \leq 2$, $k = 1.66$, $k_1 = 0.15$
 $V_{rd,c(min)} = 0.001 \times (0.37) \times 250 \times 465 = 43.01 \text{ kN}$, $v_{min} = 0.035 \cdot k^{1.50} \cdot f_{ck}^{1/2} = 0.37 \text{ N/mm}^2$ (EC2 Eq.6.3N)
 $\rho_1 = 314 / (250 \times 465) = 0.0027$, $V_{rdc} = 0.001 \times [0.120 \times 1.66 \times (0.27 \times 25.00)^{0.333}] \times 250 \times 465 = 43.76 \text{ kN}$
 $V_{sd} = 51.72 \text{ kN} > V_{rdc} = 43.76 \text{ kN}$, **Vsd > Vrdc shear reinforcement is needed**

Concrete strut capacity V_{rdmax} (EC2 §6.2.3 Eq.6.9)

$V_{rdmax} = \alpha_{cw} \cdot b_w \cdot z \cdot v_1 \cdot f_{cd} / (\cot \theta + \tan \theta)$, $V_{sd} / V_{rdmax} = 0.14$, $\theta = 21.8^\circ$, $\cot \theta = 2.50$, $\tan \theta = 0.40$
 $\alpha_{cw} = 1.00$, $z = 0.9d$, $f_{ck} = 25.0 \leq 60 \text{ MPa}$, $v_1 = 0.60$, $f_{cd} = 16.67 \text{ MPa}$
 $V_{rdmax} = 0.001 \times 1.00 \times 250 \times 0.9 \times 465 \times 0.60 \times 16.67 / 2.90 = 360.8 \text{ kN}$
 $V_{sd} = 51.7 \text{ kN} < 360.8 \text{ kN} = V_{rdmax}$, the check is verified

Shear reinforcement of vertical stirrups (EC2 §6.2.3 Eq.6.8)

$V_{rds} = (A_{sw}/s) z \cdot f_{ywd} \cdot \cot \theta$, $V_{rds} = 51.72 \text{ kN}$, $z = 0.9d$, $f_{ywd} = 0.8 f_{yk} = 400.00 \text{ N/mm}^2$, $\cot \theta = 2.50$
 $A_{sw}/s = V_{rds} / (z \cdot f_{ywd} \cdot \cot 21.80^\circ) = (1.0 \text{E} + 006) \times 51.72 / (0.9 \times 465 \times 400 \times 2.50) = 124 \text{ mm}^2/\text{m}$ ($A_{sw}/s = 1.24 \text{ cm}^2/\text{m}$)
 Required shear reinforcement: ($A_{sw}/s = 1.24 \text{ cm}^2/\text{m}$)

Minimum links for shear reinforcement (EC2 §9.2.2)
 Minimum shear reinforcement ratio $\rho_{w,min}$ (EC2 Eq.9.5N)
 $\rho_{w,min} = (0.08 \times (f_{ck})^{0.5}) / f_{yk}$, $f_{ck} = 25 \text{ N/mm}^2$, $f_{yk} = 500 \text{ N/mm}^2$, $\rho_{w,min} = 0.0008$
 $\min A_{sw}/s = 10 \times 0.0008 \times 250 \times \sin(90^\circ) = 2.00 \text{ cm}^2/\text{m}$
 Maximum longitudinal spacing of links $s_{lmax} = 0.75d(1 + \cot 90^\circ) = 345 \text{ mm}$ (EC2 §9.2.2.6, Eq.9.6N)
 Maximum transverse spacing of link legs $s_{tmax} = 0.75d$ ($\leq 600 \text{ mm}$) = 345mm (EC2 §9.2.2.8, Eq.9.8N)
 Minimum shear reinforcement stirrups $\emptyset 8/34.5$ ($A_{sw}/s = 2.92 \text{ cm}^2/\text{m}$)

Span-2 right Shear reinforcement: stirrups $\emptyset 8/34.5$ ($A_{sw}/s = 2.92 \text{ cm}^2/\text{m}$)

5.6. Serviceability limit state, crack control (EC2 EN1992-1-1:2004, §7.3.2, §7.3.3)

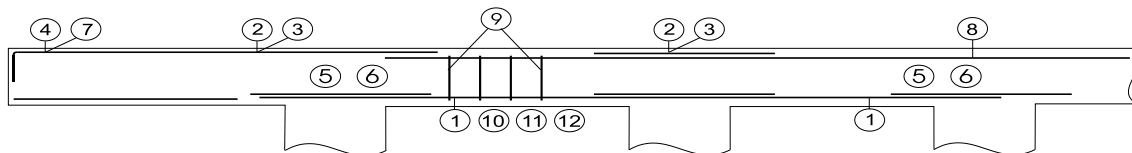
Minimum reinforcement areas $A_{s,min} = k_c \cdot k \cdot f_{ct,eff} \cdot A_{ct} / \sigma_s$ (EC2 Eq.7.1)
 $b = 0.250 \text{ m}$, $b_{eff} = 0.250 \text{ m}$, $h = 0.500 \text{ m}$, $d = 0.465 \text{ m}$, $N = 0.00 \text{ kN}$, $\sigma_c = (N/bh) = 0.00 \text{ N/mm}^2$, $\Phi = 10 \text{ mm}$
 $\max(h, b_l) = 500 \text{ mm}$, $f_{ctm} = 2.60 \text{ N/mm}^2$, $h_{c,eff} = 2.50 \times (h - d) = 87 \text{ mm}$, $k = 0.86$, $k_c = 0.40$ (EC2 Eq.7.2)
 Min. reinf. without control of crack width, $A_{s,min} = 0.40 \times 0.86 \times 2.60 \times 250 \times 87 / 500 = 39 \text{ mm}^2 = 0.39 \text{ cm}^2$
 Crack control for crack width $w_k = 0.3 \text{ mm}$, using steel diameter $\emptyset = 10 \text{ mm}$
 $\emptyset_s = \emptyset^* \cdot s (f_{ctm} / 2.9) [k_c \cdot h_{cr} / 2(h - d)]$, $\emptyset_s = 10 \text{ mm}$, $\emptyset^* = 8 \text{ mm}$, ($f_{ctm} = 2.60$, $h_{cr} = 250 \text{ mm}$) (EC2 Eq.7.6N)
 Steel bar diameter $\Phi^* = 8 \text{ mm}$, crack width $w_k = 0.3 \text{ mm}$, steel stress $\sigma_s = 360 \text{ N/mm}^2$ (EC2 Table 7.2N)
 Min. reinforcement for $w_k = 0.3 \text{ mm}$ and $\emptyset = 10 \text{ mm}$, $A_{s,min} = 0.40 \times 0.86 \times 2.60 \times 250 \times 87 / 360 = 54 \text{ mm}^2 = 0.54 \text{ cm}^2$

5.7. Serviceability limit state, deflection control (EC2 EN1992-1-1:2004, §7.4.2)

Span/effective depth, must be $L/d \leq \text{limit of EC2 Table 7.4N}$
 Span-1, $K = 1.30$, $\rho = 0.274\%$, $L/d = 3600 / 465 = 7.74 < 26.00$ (EC2 T.7.4N)
 Span-2, $K = 1.30$, $\rho = 0.274\%$, $L/d = 3600 / 465 = 7.74 < 26.00$

5.8. Reinforcing bar schedule

Num	type	reinforcing bar [mm]	items	\emptyset	g/m [kg/m]	length [m]	weight [kg]
Total weight [kg]							0.00



6. BEAM-006

Moment capacity of beam section

(EC2 EN1992-1-1:2004, EC0 EN1990-1-1:2002)

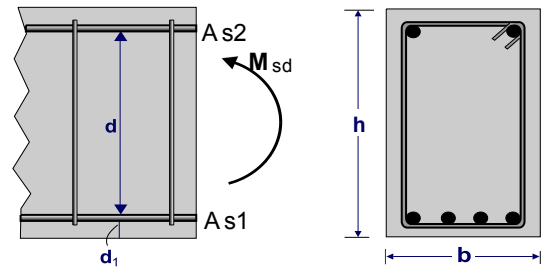
bw=0.250 m, h =0.500 m

As1=4Ø14 (6.16cm²), As2=2Ø14 (3.08cm²)

Concrete-Steel class: C25/30-S500 (EC2 §3)

Concrete cover : Cnom=20 mm (EC2 §4.4.1)

yc=1.50, ys=1.15 (EC2 Table 2.1N)



6.1. Dimensions and loads

Beam cross section bw=0.250 m, h=0.500 m

Bottom reinforcement 4Ø14 (6.16cm²)

Top reinforcement 2Ø14 (3.08cm²)

Effective depth of cross section d1=Cnomc+Øs+0.5Ø=20+8+0.5x14=35mm, d=500-35=465mm

6.2. Cross section moment capacity

(EC2 EN1992-1-1:2004, §6.1)

(iterations:12). From internal force equilibrium we have:

$\epsilon_c=3.50(o/o)$, $F_c=\alpha \cdot 0.85f_{cd} \cdot b \cdot x$, $\alpha=0.810$, $x=60.7\text{mm}$, $x/d=0.13$

$F_c=-\alpha \cdot 0.85f_{cd} \cdot b \cdot x = 0.001 \times 0.810 \times 0.85 \times 16.67 \times 250 \times 60.7 = -174\text{kN}$

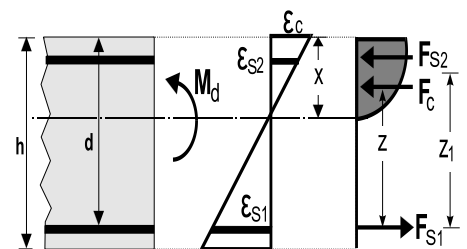
$\epsilon_{s1}=20.00(o/o) > 2.17=\epsilon_y$, $F_{s1}=As1 \cdot f_{yd}=0.001 \times 616 \times 435.0 = 268\text{kN}$

$\epsilon_{s2}=1.48(o/o) < 2.17=\epsilon_y$, $F_{s2}=-As2 \cdot E_s \cdot \epsilon_{s2} = 308 \times 200 \times 1.48 = -91\text{kN}$

$z=d-K_a \cdot x$, $K_a=0.416$, $z=465-0.416 \times 60.75=436\text{mm}$

$z_1=(zF_c+(d-d_2)F_{s2})/(F_c+F_{s2})=(436 \times 174+430 \times 91)/(174+91)=434\text{mm}$

Moment capacity of cross section $M_d=z_1 \cdot F_{s1}=0.434 \times 268 = 116.31\text{kNm}$



Ultimate moment capacity of beam cross section Md= 116.31 kNm

7. BEAM-007

Moment capacity of beam section with FRP strengthening

(EC2 EN1992-1-1:2004, EC0 EN1990-1-1:2002)

bw=0.250 m, h =0.500 m

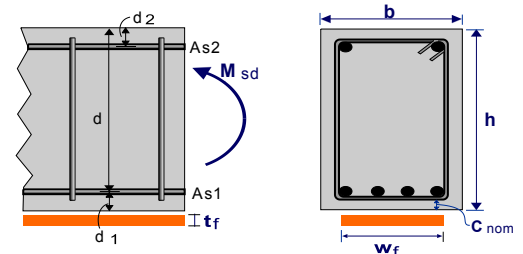
As1=4Ø14 (6.16cm²), As2=2Ø14 (3.08cm²)

FRP+epoxy, t(FRP)= 1.00 mm

Concrete-Steel class: C25/30-S500 (EC2 §3)

Concrete cover : Cnom=20 mm (EC2 §4.4.1)

yc=1.50, ys=1.15 (EC2 Table 2.1N)



7.1. Dimensions and loads

Beam cross section bw=0.250 m, h=0.500 m

Bottom reinforcement 4Ø14 (6.16cm²)

Top reinforcement 2Ø14 (3.08cm²)

Effective depth of cross section d1=Cnomc+Øs+0.5Ø=20+8+0.5x14=35mm, d=500-35=465mm

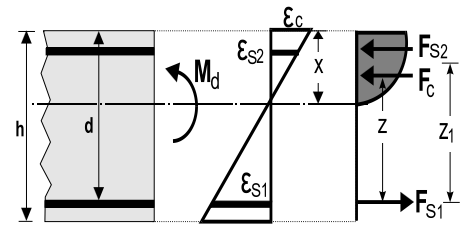
Fibre Reinforced Polymer material (FRP)

- Characteristic name : FRP+epoxy
- Total thickness : 1.00 mm
- Modulus of elasticity : 100 GPa
- Tensile strength : 1000 MPa
- Cross section area : 250x1.00= 250 mm²

7.2. Cross section moment capacity, without FRP strengthening

(EC2 EN1992-1-1:2004, §6.1)

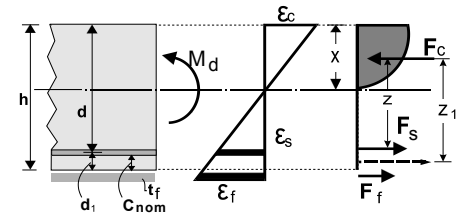
(iterations:12). From internal force equilibrium we have:
 $\epsilon_c=3.50(o/oo)$, $F_c=\alpha \cdot 0.85f_{cd} \cdot b \cdot x$, $\alpha=0.810$, $x=60.7\text{mm}$, $x/d=0.13$
 $F_c = -\alpha \cdot 0.85f_{cd} \cdot b \cdot x = 0.001 \times 0.810 \times 0.85 \times 16.67 \times 250 \times 60.7 = -174\text{kN}$
 $\epsilon_{s1}=20.00(o/oo) > 2.17=\epsilon_y$, $F_{s1} = A_{s1} \cdot f_{yd} = 0.001 \times 616 \times 435.0 = 268\text{kN}$
 $\epsilon_{s2} = 1.48(o/oo) < 2.17=\epsilon_y$, $F_{s2} = -A_{s2} \cdot E_s \cdot \epsilon_{s2} = 308 \times 200 \times 1.48 = -91\text{kN}$
 $z = d - K_a \cdot x$, $K_a = 0.416$, $z = 465 - 0.416 \times 60.75 = 436\text{mm}$
 $z_1 = (z F_c + (d - d_2) F_{s2}) / (F_c + F_{s2}) = (436 \times 174 + 430 \times 91) / (174 + 91) = 434\text{mm}$
 Moment capacity of cross section $M_d = z_1 \cdot F_s = 0.434 \times 268 = 116.31\text{kNm}$



7.3. Cross section moment capacity, with FRP strengthening

(EC2 EN1992-1-1:2004, §6.1)

(iterations:5). From internal force equilibrium we have:
 $\epsilon_c=3.50(o/oo)$, $F_c=\alpha \cdot 0.85f_{cd} \cdot b \cdot x$, $\alpha=0.810$, $x=132.0\text{mm}$, $x/d=0.28$
 $F_c = -\alpha \cdot 0.85f_{cd} \cdot b \cdot x = 0.001 \times 0.810 \times 0.85 \times 16.67 \times 250 \times 132.0 = -378\text{kN}$
 $\epsilon_{s1} = 8.83(o/oo) > 2.17=\epsilon_y$, $F_{s1} = A_{s1} \cdot f_{yd} = 0.001 \times 616 \times 435.0 = 268\text{kN}$
 $\epsilon_{s2} = 2.57(o/oo) > 2.17=\epsilon_y$, $F_{s2} = -A_{s2} \cdot f_{yd} = 0.001 \times 308 \times 435.0 = -134\text{kN}$
 $\epsilon_f + \epsilon_{fo} = 9.78(o/oo)$, $\epsilon_f = 9.78$, $\sigma_f = E_f \cdot \epsilon_f = 100 \times 9.78 = 978\text{MPa}$
 $\sigma_f = 978\text{MPa} < 1000$ (tensile strength) $F_f = A_f \cdot \sigma_f = 250 \times 978 = 244\text{kN}$
 $z = d - K_a \cdot x$, $K_a = 0.416$, $z = 465 - 0.416 \times 131.95 = 410\text{mm}$
 $z_1 = ((d - z - d_2) \cdot F_{s2} + (d_1 + t_f / 2) \cdot F_f) / (F_s + F_f) + z = (20 \times 134 + 36 \times 244) / 512 + 410 = 432\text{mm}$
 Moment capacity of cross section $M_d = z_1 \cdot (F_s + F_f) = 0.432 \times (268 + 244) = 221.18\text{kNm}$



Ultimate moment capacity of beam cross section $M_d = 221.18\text{ kNm}$

7.4. Increase of beam shear strength

FRP strengthening jacket on the vertical beam faces of thickness 1.000 mm
 (assumed effective design strain $\epsilon_f=0.002$, shape coefficient $a=2.00$)
 $V_{sf} = a \cdot \epsilon_f \cdot E_f \cdot t_f \cdot h \cdot \cot\theta = 2.00 \times 0.002 \times 100.0 \times 1.000 \times 500 \times 1 = 200\text{kN}$, **$V_{sf} = 200\text{kN}$**

8. BEAM-008

Moment capacity of T beam section

(EC2 EN1992-1-1:2004, EC0 EN1990-1-1:2002)

$b_w = 0.200\text{ m}$, $h = 0.600\text{ m}$

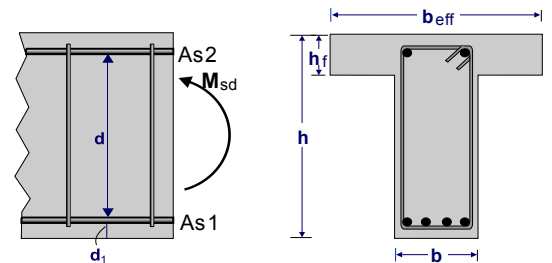
$b_{eff} = 1.250\text{ m}$, $h_f = 0.180\text{ m}$

$A_{s1} = 4\phi 14$ (6.16cm²), $A_{s2} = 2\phi 14$ (3.08cm²)

Concrete-Steel class: C25/30-S500 (EC2 §3)

Concrete cover : $C_{nom} = 25\text{ mm}$ (EC2 §4.4.1)

$\gamma_c = 1.50$, $\gamma_s = 1.15$ (EC2 Table 2.1N)



8.1. Dimensions and loads

Beam cross section $b_w = 0.200\text{ m}$, $h = 0.600\text{ m}$, $b_{eff} = 1.250\text{ m}$, $h_f = 0.180\text{ m}$

Bottom reinforcement $4\phi 14$ (6.16cm²)

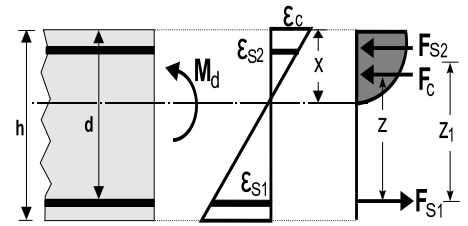
Top reinforcement $2\phi 14$ (3.08cm²)

Effective depth of cross section $d_1 = C_{nom} + \phi_s + 0.5\phi = 25 + 8 + 0.5 \times 14 = 40\text{mm}$, $d = 600 - 40 = 560\text{mm}$

8.2. Cross section moment capacity

(EC2 EN1992-1-1:2004, §6.1)

T beam cross section, neutral axis within the depth of top flange $x=27.8 \leq h=180.0\text{mm}$
 (iterations:3). From internal force equilibrium we have:
 $\epsilon_c=1.43(o/o)$, $F_c=\alpha \cdot 0.85f_{cd} \cdot b \cdot x$, $\alpha=0.545$, $x=27.8\text{mm}$, $x/d=0.05$
 $F_c = -\alpha \cdot 0.85f_{cd} \cdot b \cdot x = 0.001 \times 0.545 \times 0.85 \times 16.67 \times 1250 \times 27.8 = -268\text{kN}$
 $\epsilon_{s1}=20.00(o/o) > 2.17 = \epsilon_y$, $F_{s1} = A_{s1} \cdot f_{yd} = 0.001 \times 616 \times 435.0 = 268\text{kN}$
 $\epsilon_{s2} = 0.00(o/o) < 2.17 = \epsilon_y$, $F_{s2} = -A_{s2} \cdot E_s \cdot \epsilon_{s2} = 308 \times 200 \times 0.00 = 0\text{kN}$
 $z = d - K_a \cdot x$, $K_a = 0.359$, $z = 560 - 0.359 \times 27.78 = 547\text{mm}$
 $z_1 = (zF_c + (d-d_2)F_{s2}) / (F_c + F_{s2}) = (547 \times 268 + 520 \times 0) / (268 + 0) = 547\text{mm}$
 Moment capacity of cross section $M_d = z_1 \cdot F_{s1} = 0.547 \times 268 = 146.60\text{kNm}$



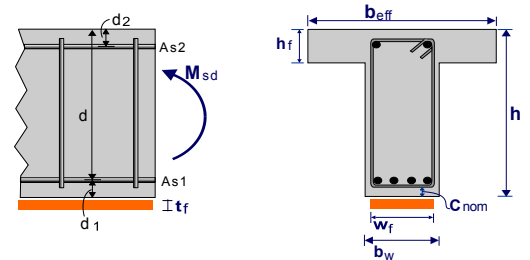
Ultimate moment capacity of beam cross section $M_d = 146.60\text{ kNm}$

9. BEAM-009

Moment capacity of T beam section with FRP strengthening

(EC2 EN1992-1-1:2004, EC0 EN1990-1-1:2002)

$b_w = 0.250\text{ m}$, $h = 0.700\text{ m}$
 $b_{eff} = 1.100\text{ m}$, $h_f = 0.200\text{ m}$
 $A_{s1} = 4\phi 14$ (6.16cm^2), $A_{s2} = 2\phi 14$ (3.08cm^2)
FRP+epoxy, $t(\text{FRP}) = 1.00\text{ mm}$
 Concrete-Steel class: C25/30-S500 (EC2 §3)
 Concrete cover : $C_{nom} = 20\text{ mm}$ (EC2 §4.4.1)
 $\gamma_c = 1.50$, $\gamma_s = 1.15$ (EC2 Table 2.1N)



9.1. Dimensions and loads

Beam cross section $b_w = 0.250\text{ m}$, $h = 0.700\text{ m}$, $b_{eff} = 1.100\text{ m}$, $h_f = 0.200\text{ m}$
 Bottom reinforcement $4\phi 14$ (6.16cm^2)
 Top reinforcement $2\phi 14$ (3.08cm^2)
 Effective depth of cross section $d_1 = C_{nom} + \phi_s + 0.5\phi = 20 + 8 + 0.5 \times 14 = 35\text{mm}$, $d = 700 - 35 = 665\text{mm}$

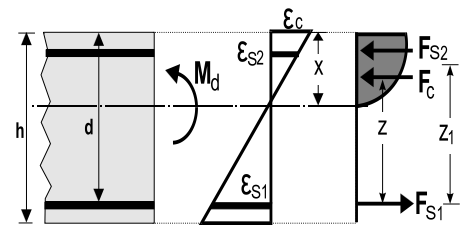
Fibre Reinforced Polymer material (FRP)

Characteristic name : FRP+epoxy
 Total thickness : 1.00 mm
 Modulus of elasticity : 100 GPa
 Tensile strength : 1000 MPa
 Cross section area : $250 \times 1.00 = 250\text{ mm}^2$

9.2. Cross section moment capacity, without FRP strengthening

(EC2 EN1992-1-1:2004, §6.1)

T beam cross section, neutral axis within the depth of top flange $x=31.6 \leq h=200.0\text{mm}$
 (iterations:3). From internal force equilibrium we have:
 $\epsilon_c=1.43(o/o)$, $F_c=\alpha \cdot 0.85f_{cd} \cdot b \cdot x$, $\alpha=0.545$, $x=31.6\text{mm}$, $x/d=0.05$
 $F_c = -\alpha \cdot 0.85f_{cd} \cdot b \cdot x = 0.001 \times 0.545 \times 0.85 \times 16.67 \times 1100 \times 31.6 = -268\text{kN}$
 $\epsilon_{s1}=20.00(o/o) > 2.17 = \epsilon_y$, $F_{s1} = A_{s1} \cdot f_{yd} = 0.001 \times 616 \times 435.0 = 268\text{kN}$
 $\epsilon_{s2} = 0.00(o/o) < 2.17 = \epsilon_y$, $F_{s2} = -A_{s2} \cdot E_s \cdot \epsilon_{s2} = 308 \times 200 \times 0.00 = 0\text{kN}$
 $z = d - K_a \cdot x$, $K_a = 0.359$, $z = 665 - 0.359 \times 31.57 = 649\text{mm}$
 $z_1 = (zF_c + (d-d_2)F_{s2}) / (F_c + F_{s2}) = (649 \times 268 + 630 \times 0) / (268 + 0) = 649\text{mm}$
 Moment capacity of cross section $M_d = z_1 \cdot F_{s1} = 0.649 \times 268 = 173.93\text{kNm}$



9.3. Cross section moment capacity, with FRP strengthening

(EC2 EN1992-1-1:2004, §6.1)

T beam cross section, neutral axis within the depth of top flange $x=39.2 \leq h=200.0\text{mm}$

(iterations:12). From internal force equilibrium we have:

$$\epsilon_c = 3.50 (o/o), F_c = \alpha \cdot 0.85 f_{cd} \cdot b \cdot x, \alpha = 0.810, x = 39.2\text{mm}, x/d = 0.06$$

$$F_c = -\alpha \cdot 0.85 f_{cd} \cdot b \cdot x = 0.001 \times 0.810 \times 0.85 \times 16.67 \times 1100 \times 39.2 = -495\text{kN}$$

$$\epsilon_{s1} = 20.00 (o/o) > 2.17 = \epsilon_y, F_{s1} = A_{s1} \cdot f_{yd} = 0.001 \times 616 \times 435.0 = 268\text{kN}$$

$$\epsilon_{s2} = 0.38 (o/o) < 2.17 = \epsilon_y, F_{s2} = -A_{s2} \cdot E_s \cdot \epsilon_{s2} = 308 \times 200 \times 0.38 = -23\text{kN}$$

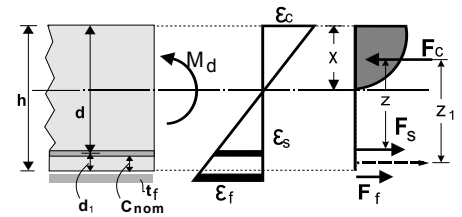
$$\epsilon_f + \epsilon_{fo} = 10.00 (o/o), \epsilon_f = 10.00, \sigma_f = E_f \cdot \epsilon_f = 100 \times 10.00 = 1000\text{MPa}$$

$$\sigma_f = 1000\text{MPa} < 1000 (\text{tensile strength}), F_f = A_f \cdot \sigma_f = 250 \times 1000 = 250\text{kN}$$

$$z = d - K_a \cdot x, K_a = 0.416, z = 665 - 0.416 \times 39.21 = 624\text{mm}$$

$$z_1 = ((d - z - d_2) \cdot F_{s2} + (d_1 + t_f/2) \cdot F_f) / (F_s + F_f) + z = (6 \times 23 + 36 \times 250) / 518 + 624 = 641\text{mm}$$

$$\text{Moment capacity of cross section } M_d = z_1 \cdot (F_s + F_f) = 0.641 \times (268 + 250) = 332.04\text{kNm}$$



Ultimate moment capacity of beam cross section $M_d = 332.04 \text{ kNm}$


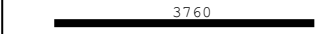
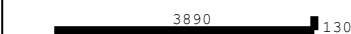

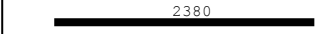
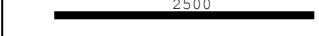

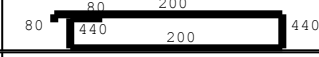
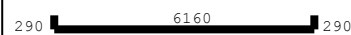




9.4. Increase of beam shear strength

FRP strengthening jacket on the vertical beam faces of thickness 1.000 mm

(assumed effective design strain $\epsilon_f = 0.002$, shape coefficient $a = 2.00$)

$$V_{sf} = a \cdot \epsilon_f \cdot E_f \cdot t_f \cdot h \cdot \cot \theta = 2.00 \times 0.002 \times 100.0 \times 1.000 \times 500 \times 1 = 200\text{kN}, \quad \mathbf{V_{sf} = 200\text{kN}}$$

Reinforcing bar schedule

Num	Structure object	type	reinforcing bar [mm]	items	∅	g/m [kg/m]	length [m]	weight [kg]
1	BEAM-005 (Span-1)	B11	130  3890	4	10	0.617	4.020	9.92
2	BEAM-005 (Span-1)	B8	 3760	2	10	0.617	3.760	4.64
3	BEAM-005 (Span-2)	B12	 3890 130	4	10	0.617	4.020	9.92
4	BEAM-005 (Span-2)	B8	 3760	2	10	0.617	3.760	4.64
5	BEAM-005 (Supp-1)	B2	 2380	4	10	0.617	2.380	5.87
6	BEAM-005 (Supp-1)	B2	 2500	1	12	0.888	2.500	2.22
7	BEAM-005 (Span-1)	B9	80  440 200 440	10	8	0.395	1.440	5.69
8	BEAM-005 (Span-2)	B9	80  440 200 440	10	8	0.395	1.440	5.69
9	BEAM-004 (Span-1)	B10	290  6160 290	4	18	2.000	6.740	53.92
10	BEAM-004 (Span-1)	B10	330  6160 330	2	20	2.470	6.820	33.69
11	BEAM-004 (Span-1)	B8	 6160	2	14	1.210	6.160	14.91
12	BEAM-004 (Span-1)	B9	80  420 200 420	21	8	0.395	1.400	11.61
13	BEAM-004 (Cant2)	B9	80  440 200 440	0	8	0.395	1.440	0.00
Total weight [kg]								162.72